MAT 4199/5145 – Fall 2017

Assignment 4

Professor: Alistair Savage

Due date/time: November 1, 2017, 3pm

Assignments should be placed in the box for this course found in the lobby of the Department of Mathematics & Statistics (585 King Edward Ave) or handed in before/after class prior to the due date/time. No assignments will be accepted after the department receptionist has closed the box. Therefore, you should plan to submit your assignment well in advance of the due date/time. Please clearly indicate your name and student number on each page of your assignment. Assignments of more than one page that are not stapled will have 20% of the total available points deducted from their grade. You must properly acknowledge any collaboration or help. See

alistairsavage.ca/mat5145/assignments

for further details.

QUESTION 1 **(4 pts)**. [Exercise 1.5.10] Show that (G, K) is a symmetric Gelfand pair if and only if $g^{-1} \in KgK$ for all $g \in G$. Note that this corresponds to the case $\tau = I_G$ in Proposition 1.5.13.

QUESTION 2 **(4 pts)**. [Exercise 1.5.11] A group G is *ambivalent* if g^{-1} is conjugate to g for every $g \in G$. Consider the action of $G \times G$ on G by

$$(g_1, g_2) \cdot g = g_1 g g_2^{-1}, \quad \text{for all } g_1, g_2, g \in G.$$

Let $\tilde{G} = \{(g, g) : g \in G\}$. Show that the Gelfand pair $(G \times G, \tilde{G})$ is symmetric if and only if G is ambivalent.

QUESTION 3 **(6 pts)**. [Exercise 1.6.1]

(a) Prove that f_v, as defined in (1.49), is an element of Z.
(b) Prove that $\tilde{V}$, as defined in (1.50), is a K-invariant subspace of Z.
(c) Prove that the map $V \to \tilde{V}$, $v \mapsto f_v$, is an isomorphism of K-representations.
(d) Prove that (1.52) is the unique way to write $f \in Z$ as a sum of elements of $\sigma(s)\tilde{V}$, $s \in S$. *Hint*: f is uniquely determined by its values on S.

QUESTION 4 **(2 pts)**. [Exercise 1.6.3] Prove that $\check{U}$, as defined in the proof of Theorem 1.6.6, is an element of $\operatorname{Hom}_G\left(W, \operatorname{Ind}_K^G V\right)$.

QUESTION 5 **(4 pts)**. [Exercise 1.6.6]

(a) Prove that Z_s, as defined in the proof of Theorem 1.6.9, is H-invariant.
(b) Verify that the maps (1.56) and (1.57) defined in the proof Theorem 1.6.9 are mutually inverse, and that (1.56) intertwines the H-action.