University of Ottawa

Department of Mathematics and Statistics

MAT 3543 : Théorie des Anneaux
Professor : Hadi Salmasian

Midterm Exam

February 28, 2014

Surname _____ First Name _____

Student # _____

Instructions :

(a) You have 80 minutes to complete this exam.

(b) The number of points available for each question is indicated in square brackets.

(c) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.

(d) Write your student number at the top of each page in the space provided.

(e) No notes, books, scrap paper, calculators or other electronic devices are allowed.

(f) You are strongly recommended to write in **pen**, not pencil.

(g) You may use the last page of the exam as scrap paper.

Good luck !

Please do not write in the table below.

Question	1	2	3	4	Total
Maximum	6	7	4	5	22
Grade					

1.

(a) **[2 pt]** Write down the definition of a principal ideal in a ring R.

(b) **[1 pt]** Write down the definition of a simple ring.

(c) **[1 pt]** Write down the definition of a division algebra.

(d) **[1 pts]** Give an example of a simple ring which is not a division algebra.
(It is not necessary to prove that your example satisfies the above properties.)

(e) **[1 pts]** True or false ?

- Let R and S be two simple rings. Then $R \times S$ is a simple ring.

You should justify your response.

2. (a) **[3 pts]** Determine the number of invertible elements of $\mathbb{Z}_{88}$. You should justify your response.

(b) **[1 pt]** Is the polynomial $x^4 - 15x + 10$ irreducible in $\mathbb{Q}[x]$? Justify your answer.

(c) **[1 pt]** Is the polynomial $x^4 - 15x + 10$ irreducible in $\mathbb{R}[x]$? Justify your answer.

(d) **[2 pts]** Find all of the rational roots of $x^4 + 7x - 2$, if they exist. You should justify your answer.

3. **[4 pts]** Determine all of the ideals of the ring $\mathbb{Z}_4 \times \mathbb{C}$ (direct product). Which of these ideals are prime ? You should justify your answers.

4. (a) **[3 pts]** Write down the proof of Gauss' Lemma :
Let $f(x) \in \mathbb{Z}[x]$. Suppose that $f(x) = g(x)h(x)$ where $g(x), h(x) \in \mathbb{Z}[x]$. Let p be a prime number which divides all of the coefficients of $f(x)$. Then, at least one of the following statements holds.

- p divides all of the coefficients of $g(x)$.
- p divides all of the coefficients of $h(x)$.

(b) **[2 pts]** Write down the proof of the following theorem :

– Let R be a commutative ring and I be an ideal of R such that R/I is an integral domain. Then, I is a prime ideal of R.

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