University of Ottawa
Department of Mathematics and Statistics

MAT 3341: Applied Linear Algebra

Professor: Alistair Savage

Midterm Test
June 19, 2019

Surname ______________________ First Name ______________________

Student # ______________________

Instructions:

(a) You have 80 minutes to complete this exam.
(b) Unless otherwise indicated, you must justify your answers to receive full marks.
(c) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(d) Write your student number at the top of each page in the space provided.
(e) No notes, books, scrap paper, calculators or other electronic devices are allowed.
(f) You should write in *pen*, not pencil.
(g) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	7	Total
Maximum	3	2	3	4	4	6	6	28
Grade								

QUESTION 1. **(3 points)** Find an LU factorization of

$$A = \begin{bmatrix} 0 & -1 & -1 & -2 & 1 \\ 0 & 2 & 3 & 4 & 1 \\ 0 & -3 & -1 & -6 & 7 \end{bmatrix}.$$

QUESTION 2. **(2 points)** Suppose that A, B, and C are matrices. Furthermore, suppose that B is a left inverse to A, and that C is a right inverse to A. Show that $B = C$.

QUESTION 3. **(3 points)**
(a) Show that $\|A\|_1 = \|A^T\|_\infty$ for all $A \in M_{m,n}(\mathbb{F})$.

(b) Show that $\|A\|_1 = \|A\|_\infty$ for all symmetric matrices A.

QUESTION 4. **(4 points)**

(a) Give the definition of the *conjugate transpose* A^H of a matrix $A \in M_{m,n}(\mathbb{C})$.

(b) Give the definition of a *hermitian* matrix.

(c) Give the definition of a *unitary* matrix.

(d) Prove that the product of two unitary matrices is unitary.

QUESTION 5. **(4 points)** For each of the following, either given an example, or state that no such example exists.

(a) A square matrix A such that all the eigenvalues of A are real, but A is not hermitian.

(b) A hermitian matrix A with at least one eigenvalue that is not real.

(c) A positive definite matrix that does not have a Cholesky factorization.

(d) A square matrix that is not diagonalizable.

QUESTION 6.

(a) **(3 points)** Find the condition number of the matrix

$$A = \begin{bmatrix} 1 & -1 \\ 3 & 5 \end{bmatrix}$$

with respect to the 1-norm.

(b) **(3 points)** Suppose an invertible matrix $A \in M_{3,3}(\mathbb{R})$ has condition number $\kappa(A) = 2$ with respect to the 1-norm. Furthermore, suppose that

$$A\mathbf{x} = \mathbf{b}, \quad \text{where } \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

If

$$A\mathbf{x}' = \mathbf{b}' \quad \text{where } \mathbf{b}' = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix},$$

find an upper bound on

$$\|\Delta\mathbf{x}\|_1 = \|\mathbf{x}' - \mathbf{x}\|_1.$$

In other words, find a positive real number C such that $\|\Delta\mathbf{x}\|_1 \leq C$.

QUESTION 7. **(6 points)** Consider the matrix

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 1 \end{bmatrix}.$$

Find a unitary matrix U and a diagonal matrix D such that $U^H A U = D$.

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