



# Université d'Ottawa • University of Ottawa

Faculté des sciences  
Mathématiques et de statistique

Faculty of Science  
Mathematics and Statistics

**MAT 3143 – The Midterm Test**

**Instructor: K. Zaynullin**

Last name: \_\_\_\_\_

First name: \_\_\_\_\_

Student number: \_\_\_\_\_

Please, read the following instructions carefully:

- You have 80 minutes to complete this exam. Read each question carefully. Where it is possible to check your work, do so.
- Answer all questions in the space provided after each question. You can also use the backs of the pages.
- This is a closed book exam, and no notes of any kind are allowed. The use of calculators, cell phones, pagers or any text storage or communication device is not permitted.

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THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	5	6	Total
Mark							
Out of	3	2	3	2	2	3	15

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1. Let  $R$  be a ring and let  $X$  be a nonempty subset of  $R$ . Determine whether the subset

$$\{y \in R \mid yx - xy = 0 \text{ for all } x \in X\}$$

is a subring of  $R$ . Justify your answer.

(3)

(a correct answer without any justification is 1/2 pts).

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Solution:

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**2.** Determine whether the matrix ring  $M_2(\mathbb{Z})$  is a domain. (2)

(a correct answer without any justification is 1/2 pts).

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Solution:

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**3.** Give an example of a ring  $R$  and of an ideal  $I$  in  $R$  such that the factor ring  $R/I$  is commutative but the ring  $R$  is non-commutative. Justify your answer. (3)

a correct example without verification (that  $R/A$  is commutative and  $R$  is not) gives 1 point only

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Solution:

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4. Prove that there is no ring homomorphism  $\mathbb{C} \rightarrow \mathbb{R}$ . (2)

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Solution:

5. Find a monic polynomial  $h$  in  $\mathbb{Z}_5[x]$  such that

$$(h) = \{f \in F[x] \mid f(0) = f(1) = 0\}.$$

(2)

(a correct answer without justification is 1/2pts)

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Solution:

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**6.** Let  $F$  be a field. Show that  $F[x]/(x^2 - x)$  is isomorphic (as a ring) to  $F \times F$ . (3)

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Solution: