Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 3143 – The final exam **Instructor: K. Zaynullin**

Last name: ______________________________

First name: ______________________________

Signature: ______________________________

Student number:

Please, read the following instructions carefully:

- You have 3 hours to complete this exam. Read each question carefully. Where it is possible to check your work, do so.
- Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.
- By signing, you acknowledge that you have ensured that you are complying with the above statement.

THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	5	6	7	8	9	10	11	Total
Mark												
Out of	2	3	2	2	3	2	2	2	2	3	2	25

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1. Let $R = M_2(\mathbb{C})$ be the ring of 2×2-matrices with complex entries. Verify that

$$S = \{\begin{pmatrix} x & iy \\ 0 & x \end{pmatrix} \mid x, y \in \mathbb{C}\}$$

is a subring of R (apply the subring test). (2)

Solution:

2. In each case decide whether I is an ideal of the ring R

(1) $R = \mathbb{Z}_3 \times \mathbb{Z}_3$, $I = \{(n, n + \bar{3}) \mid n \in \mathbb{Z}_3\}$ (here $\mathbb{Z}_3$ denotes integers modulo 3). (1)

(2) $R = \mathbb{Q}(i)$, $I = \{r - ri \mid r \in \mathbb{Q}\}$. (1)

(3) $R = \mathbb{R}[x]$, $I = \{p(x) \in \mathbb{R}[x] \mid p(1) = 1\}$. (1)

Solution:

3. Let F be a field of order 5. Divide $x^2 + x + 3$ by $x + 2$ in $F[x]$. (2)

Solution:

4. Determine whether $(1 + 7i)$ is irreducible in the ring $\mathbb{Z}(i)$ of Gaussian integers. (2)

Solution:

5. Prove that there is no field F such that $(F^{\times}, \cdot) \simeq (\mathbb{Z}, +)$. (3)

Solution:

6. If $a^2 = b^2$ and $a^3 = b^3$ in a domain, show that $a = b$. (2)

Solution:

7. Determine the multiplicity of a as a root of $f(x) \in F[x]$, where
$f(x) = x^4 + 2x^2 + 2x + 2$, $a = -1$ and $F = \mathbb{Z}_3$. (2)

Solution:

8. Let F be a field. Find a monic polynomial h in $F[x]$ such that (2)

$$(h) = \{f \in F[x] \mid \text{ the sum of the coefficients of } f \text{ is zero}\}.$$

Solution:

9. If $u \in \mathbb{C}$ and $u \notin \mathbb{R}$, show that $\mathbb{C} = \mathbb{R}(u)$. (2)

Solution:

10. Find a basis of $\mathbb{Q}(\sqrt{3}, \sqrt[3]{3})$ over $\mathbb{Q}$. (3)

Solution:

11. Show that $x^2 - 3$ and $x^2 - 2x - 2$ have the same splitting fields. (2)

Solution: