University of Ottawa

Department of Mathematics and Statistics

MAT 3143: Algebraic Structures II

Professor Erhard Neher

Midterm (February 24, 2009)

Family Name: ______________________

First Name: ______________________

Student number: ______________________

Instructions:

- The test has 4 questions, maximum 42 points. You have 80 minutes to complete the exam.
- No books, notes or calculators are allowed. Cellular phones or any other electronic devices are not allowed.
- The exam has 5 unstapled pages. If you do not have enough space, use spare pages (distributed).
- Before you hand in the exam, please put the pages in the right order. They will be stapled.

Quest.	1.	2.	3.	4.	Total
maximal	10	12	10	10	42
score					

Question 1: (10 points) (a) State the definition of (i) a maximal ideal of a ring R, and (ii) a simple ring.

(b) Let I be an ideal of a ring R. Prove that I is a maximal ideal of R if and only if R/I is a simple ring.

Question 2: (12 points) (a) State the definition of a general ring homomorphism and a ring homomorphism.

(b) Show that every surjective ring homomorphism is a ring homomorphism.

(c) Determine all ring homomorphisms $\mathbb{Z}_8 \to \mathbb{Z}_4$.

Question 3: (10 points) State the theorem on the existence and properties of the greatest common divisor of two polynomials f, $g \in F[x]$ for F an arbitrary field, and prove it.

Question 4 (10 points) Give an example of [no justification required]:

(a) a subring of $\mathrm{M}_2(\mathbb{C})$, the ring of complex 2×2 matrices;

(b) a ring which is not an integral domain;

(c) a prime ideal which is not a maximal ideal;

(d) an irreducible polynomial in $F[x]$, where F is an arbitrary field;

(e) an irreducible polynomial of degree ≥ 10 in $\mathbb{Q}[x]$.