University of Ottawa
Department of mathematics and statistics

MAT 3141 - MIDTERM EXAMINATION **Instructor: K. Zaynullin**

Last name (IN CAPITAL LETTERS) ____________________

First name ____________________

Signature ____________________

Student number ____________________

Read the following instructions:

- Not allowed: textbooks, course notes, electronic devices (calculators, telephones)
- Use only a black/blue pen to write the solution.
- Do not detach the pages of this examination.
- You may use the back of the pages as scrap paper for calculations, or to answer questions if you run out of space on the front side.
- All questions are long answer questions. You must give clear and complete solutions, with calculations, explanations and justifications. Make sure that your answer is clearly indicated.

THIS SPACE IS RESERVED FOR THE MARKER:

Question	1	2	3	4	Total
Mark					
Out of	5	7	8	10	30

1. Let V be a vector space over $\mathbb{Q}$ with basis $\{v_1, v_2\}$ and let T and S be linear operators on V satisfying

$$T(v_1 - v_2) = v_1 + v_2,\ T(v_1 + v_2) = v_1 - v_2,$$

$$S(v_1) = v_1 + v_2,\ S(v_2) = v_1 - v_2.$$

Compute $\quad (2T + S)^{\circ 2}(v_1 + v_2)$ (5)

2. Let $V = \langle 1, x, x^2, x^3 \rangle_{\mathbb{R}}$ be the vector space over $\mathbb{R}$ of polynomials of degree at most 3.

(a) Show that the map $T\colon V \to \mathbb{R}$ given by

$$T(p) = \int_0^1 p'(x)x \, dx$$

for each $p \in V$ is a linear map (here p' denotes the derivative of p). (2)

(b) Compute the transformation matrix of T with respect to the bases $\{1, x, x^2, x^3\}$ of V and $\{1\}$ of $\mathbb{R}$. (5)

(see the next page)

3. Let

$$p_1(x) = x^3 + \bar{1} \quad \text{and} \quad p_2(x) = x^5 + \bar{1}$$

be two polynomials in $\mathbb{Z}/2\mathbb{Z}[x]$.

(a) Find the gcd of p_1 and p_2. (4)

(b) Express the gcd of p_1 and p_2 as a linear combination of p_1 and p_2. (4)

(see the next page)

4. For the following matrix over $\mathbb{Q}[x]$

$$U = \begin{pmatrix} x^2 - 1 & 0 & x - 1 \\ x^2 + x - 2 & x^3 - 1 & 0 \end{pmatrix}$$

(a) Find the elementary divisors of U. (5)

(b) Find the invertible matrices P and Q such that PUQ is in canonical form. (5)

(this is the last page)