MAT 2125 – Winter 2017

Quiz 2 – Solution

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QUESTION **(4 pts)**. Is the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$$

convergent? Is it absolutely convergent?

Solution: For all $n \in \mathbb{N}$, we have

$$n+1 > n \implies 2(n+1) > 2n \implies 2(n+1)+3 > 2n+3 \implies \frac{1}{2n+2} > \frac{1}{2(n+1)+3} > 0.$$

So the absolute values of the terms of the series are positive and decreasing. Also,

$$\frac{1}{2n+3} \to 0 \quad \text{as} \quad n \to \infty.$$

Thus, the series converges by the alternating series test.

Now,

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{2n+3} \right| = \sum_{n=1}^{\infty} \frac{1}{2n+3}.$$

For all $n \in \mathbb{N}$, we have $n \geq 1$, and so

$$\frac{1}{2n+3} \geq \frac{1}{2n+3n} = \frac{1}{5n}.$$

Since the series

$$\sum_{n=1}^{\infty} \frac{1}{5n}$$

diverges (it is a multiple of the harmonic series), the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+3}$$

is *not* absolutely convergent.