MAT 2125 – Winter 2017

Quiz 1 – Solution

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QUESTION **(3 pts)**. Using the definition of a limit, prove that if $\{a_n\}$ is bounded, then $\{a_n/n\}$ converges.

Solution: Suppose $\{a_n\}$ is bounded. Then there exists M such that

$$|a_n| \leq M \quad \forall\, n \in \mathbb{N}.$$

Let $\varepsilon > 0$. Choose $N \in \mathbb{N}$ such that

$$N > \frac{M}{\varepsilon}.$$

Then, for $n \geq N$, we have

$$\left|\frac{a_n}{n}\right| = \frac{|a_n|}{n} \leq \frac{M}{N} < \frac{M}{M/\varepsilon} = \varepsilon.$$

So $a_n/n \to 0$.