MAT 2122 – Fall 2021

Midterm Exam

Professor: Alistair Savage

Your solutions should be submitted through Brightspace in ***.pdf, .jpg, or .png format****. It is your responsibility to make sure that your handwriting is legible and that your scan is of high enough quality that it can be easily read. You should always justify your answer, unless otherwise specified.*

This exam has a possible oral component. You may be contacted after the test to arrange a Zoom meeting to explain your solutions. If you are contacted, these explanations are a part of your midterm test, and will be taken into account when determining your grade.

This exam ends at 2:15pm. You may not write anything on your pages after this time. You will then have until 2:25pm to scan and submit your solutions on Brightspace.

QUESTION 1 **(7 pts)**. Consider the function

$$f\colon \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = e^{y-x}\cos y.$$

(a) Find the degree 1 Taylor polynomial $p_1(h_1,h_2)$ of f at the point $(0,0)$.
(b) Find the degree 2 Taylor polynomial $p_2(h_1,h_2)$ of f at the point $(0,0)$.

QUESTION 2 **(5 pts)**. Is the equation

$$y^2z^3 = e^{xyz}$$

solvable for $z = g(x,y)$ near the point $(x,y,z) = (0,-1,1)$? If so, compute $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ at $(x,y) = (0,-1)$.

QUESTION 3 **(5 pts)**. Define $g\colon \mathbb{R}^3 \to \mathbb{R}^3$ by

$$g(x,y,z) = \left(e^{xy}, x^2+y, x\sin z\right).$$

Suppose also that $f\colon \mathbb{R}^3 \to \mathbb{R}$ is a differentiable function with

$$f(-1,0,\pi) = -2, \quad f(1,1,0) = 5, \quad \nabla f(-1,0,\pi) = (2,1,0), \quad \text{and} \quad \nabla f(1,1,0) = (5,1,3).$$

Compute $\nabla(f\circ g)(-1,0,\pi)$.

QUESTION 4 **(8 pts)**. Determine whether the function

$$f(x,y) = x^2+y^2-2x-4y$$

has a minimum and maximum on the set

$$S = \{(x,y) : x^2+y^2 \le 20\}.$$

If so, determine the minimum and maximum values and the points at which they occur.

QUESTION 5 **(5 pts)**. Let

$$W = \{(x,y,z) : x^2+y^2+z^2 \le 4,\ z \ge 0\} \subseteq \mathbb{R}^3.$$

Compute

$$\iiint_W \cos\left(\left(\sqrt{x^2+y^2+z^2}\right)^3\right) dx\,dy\,dz\,.$$