uOttawa

MAT2122, Final Exam

Fall 2015.

Instructor:

Mohammad Bardestani.

Family Name: ________________________________

First Name: ________________________________

Student number: ________________________________

- Time: 3 hours.
- Only basic scientific calculators are permitted (non-graphing, non-programmable, no integration or differentiation capabilities). Notes or books are not permitted. The use of a calculator with any of these banned capabilities is considered **academic fraud**.
- **Show all work, clearly and in order**, if you want to get full credit. Points may be taken off if it is not clear how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. Points may be taken off for rambling and for incorrect or irrelevant statements.

- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- *Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam, academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.*

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature: ________________________________

Total marks: __________ out of 65

Problem	1	2	3	4	5	6	7	8	9	10
Marks										

Question 1: [**5 points**] Let $f(x,y,z) = (y - z, x^2 + y^2)$ and $g(u,v) = (\sin(uv), e^{u-v}, u + v - 1)$. Find the derivative and the Jacobian of the map $f \circ g$ at the point $(0,0)$.

Question 2: **[5 points]** Let $f(x, y) = e^{xy+2} - \ln(x + y^3)$. Find the equation of the tangent plane to the graph of f at the point $(2, -1)$. By using second order Taylor's formula find an approximate value of $f(2.01, -0.98)$.

Question 3: [**5 points**] Find and classify all critical points of the function

$$f(x, y) = x^3 - 3x + 2y^3 - 3y^2 - 12y + 1.$$

Question 4: **[6 points]** By using Lagrange multipliers find the absolute minimum and maximum of the function $f(x, y) = x^2 - y^2$ on the region D defined by $x^2 + y^2 \leq 1$.

Question 5 [8 points]: Evaluate the given integrals:

1): $\iint_R \sin(x^2 + y^2)\, dA$, where R is the region in the first quadrant between the circles with center the origin and radii 1 and 3.

2): $\iiint_E xe^{x^2+y^2+z^2}\, dV$, where E is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$, that lies in the first octant.

Question 6 [4 points] By applying an appropriate change of variables, evaluate

$$\iint_R e^{x+y}\,dA,$$

where R is given by the inequality $|x| + |y| \leq 1$.

Question 7 [8 points] Let $C = A_1A_2A_3A_4$ be the piecewise linear "Z-shaped" curve consecutively joining the points $A_1 = (0,1), A_2 = (1,1), A_3 = (0,0), A_4 = (1,0)$ on the coordinate plane. Find

$$I = \int_C xy\,dx - y^2\,dy.$$

Question 8 [8 points] Find the surface integral

$$I = \iint_S (x^2 - yz)\, d\mathbf{s},$$

where S is the sphere of radius 2 centered at the point $(0, 0, 0)$.

Question 9 [8 points] By using Stokes's formula, evaluate

$$\oint_C F \cdot d\mathbf{s},$$

where $F(x, y, z) = -y^2\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$ and C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$. (Orient C to be counterclockwise when viewed from above.)

Question 10 [8 points] By using Gauss' divergence formula find

$$\iint_S F \cdot d\mathbf{s},$$

where $F(x, y, z) = 3xy^2\mathbf{i} + xe^z\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 1$ and the planes $x = -1$ and $x = 2$.

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