MAT 1302B – Mathematical Methods II

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Course webpage:

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Basic set theory

A set is a collection of objects. These objects are called the elements of the set and can be mathematical, physical, or anything else.

Examples

1. The set of all Canadian citizens. Its members are the people that are Canadian citizens.
2. The set of all three letter English words.
3. The set of all even numbers.

The empty set

The empty set is the set that has no elements.

Example: The set of all English words with one million letters is empty.

The empty set is denoted by $\emptyset$.

Warning: The number 0 (zero) and the empty set $\emptyset$ are not the same!!

Basic set theory: Notation

We usually denote a set by braces: $\{2, 3, 5\}$ is the set with the three elements 2, 3, and 5.

We denote the set of all elements with a certain property (say property P) by

$$\{x \mid x \text{ has property } P\}$$

Examples

1. $$\{x \mid x \text{ is a Canadian citizen}\}$$

   is the set of all Canadian citizens.

2. $$\{(x, y) \mid x \text{ is a three letter English word}, \; y \text{ is an even number}\}$$

   is the set of all pairs (x, y) where x is a three letter English word and y is an even number.

Basic set theory: Notation

The symbol $\in$ is used to mean "is an element of".

So if A is a set, then $x \in A$ means that x is an element of the set A.

Example

If A is the set of even numbers. Then $x \in A$ means that x is an even number.

One element sets

Sets can have a single element.

Example: $\{4\}$ is the set that contains only the number 4.

Important

There is a difference between a single element set and the element itself.

Example: The set $\{4\}$ is not the same as the number 4 itself – the former is a set containing the number 4, whereas the latter is the number itself.

Think of the difference between a "tennis ball" and a "box containing a tennis ball".

Sets of numbers

Here are some sets that will be important in the course:

- Integers: The set of integers (positive and negative whole numbers, and zero) will be denoted $\mathbb{Z}$.
- Real numbers: The set of real numbers (any number that can be written as a decimal – infinite decimal expressions are allowed) will be denoted $\mathbb{R}$.

Note that the set $\mathbb{Z}$ is a subset of $\mathbb{R}$ (we write $\mathbb{Z} \subseteq \mathbb{R}$). That is, every integer is also a real number.

Examples

- $2 \in \mathbb{Z}$
- $-5 \in \mathbb{Z}$
- $4 \in \mathbb{R}$
- $\frac{-2}{3} \in \mathbb{R}$
- $\sqrt{2} \in \mathbb{R}$
- $\pi \in \mathbb{R}$

Logic

If P and Q are statements, then "$P \implies Q$" means that statement P implies statement Q.

Example

$$x \in \mathbb{Z} \implies x \in \mathbb{R}$$

since the statement "x is an integer" implies the statement "x is a real number" (since all integers are real numbers)

"$P \iff Q$" means that statement P implies statement Q and statement Q implies statement P – so the statements are equivalent.

Example

$$x \text{ is an even number} \iff x \text{ is an integer divisible by } 2$$

Logic

Another way of saying that the statements P and Q are equivalent is to say

"P is true if and only if Q is true"

We sometimes write "iff" for "if and only if".