University of Ottawa – MAT 1302

Markov Chain Exercises

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QUESTION 1. (2010 Final Exam) I tend to be rather moody at times. If I am in a good mood today, there is an 80% chance I will still be in a good mood tomorrow and 20% chance that I will be grumpy tomorrow. But if I am grumpy today, there is only a 60% chance that my mood will be good tomorrow.

(a) Determine the migration (transition) matrix for this situation.
(b) If I am in a good mood on Monday, what is the probability that I will be in a good mood on the coming Wednesday?
(c) In the long term, what percentage/fraction of the time am I in a good mood? *Hint:* Find the steady-state vector of the migration matrix and justify why it describes the long term behaviour.

QUESTION 2. (2009 Final Exam) Two telephone companies *OutOfOrder* and *NoService* are competing. A statistical study has shown that in each **6 month** period, 60% of the clients of OutOfOrder stay with the company while 40% of them which to NoService. During the same period, 20% of the clients of NoService switch to OutOfOrder, while 80% stay with NoService. On January 1, 2009, OutOfOrder had 50 thousand clients and NoService had 25 thousand clients.

(a) Write the migration matrix M.
(b) How many clients will each company have on July 1, 2009?
(c) How many clients will each company have on January 1, 2010?
(d) Assuming the migration matrix M stays constant in the long term, find the market share (i.e. percentage of the total customers) of each company in the long term. That is, find the equilibrium market shares.

QUESTION 3. (2008 Final Exam) In a certain country, the mobile phone industry is dominated by two companies: Ten-Fours and Siren. Ten-Fours has 180 000 customers and Siren has 120 000 customers. Assume that, every year, 10% of the customer base of Ten-Fours switches to Siren and 5% of the customer base of Siren switches to Ten-Fours. For the purposes of this question, suppose no customer leaves a company without switching to the other one and no company attracts customers that are not leaving the other (that is, the only changes in customer base come from switching between the two companies).

(a) Write down the transition (migration) matrix M and initial state vector $\mathbf{x}_0$ for this problem.
(b) Find the number of customers of Ten-Fours after one year.
(c) Find the number of customers of Ten-Fours after many years. That is, find the long term stable number of customers of Ten-Fours.