



Université d'Ottawa • University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

MAT 1302A: Mathematical Methods II

Professor : Aaron Tikuisis

Midterm 2 – V.A.

March 1, 2019

Name _____ Surname _____

Student # _____ DGD _____

Instructions:

- Write your full name and student number at the top of each page in the designated space.
- The duration of this midterm is 70 minutes.
- This is a closed book midterm that contains **6 questions**.
- No calculators are permitted.
- Use the designated space to answer each question. If ever you need additional space, you may write on the back of the page. If you do so, you must clearly indicate where the rest of your work can be found.
- Unless otherwise specified, the correct answer requires full justification written legibly and logically: you must convince me that you know your solution is correct.
- There is an additional blank page at the end of this exam that you may use as scrap paper.
- The use of cellphones, electronic devices and course notes is strictly forbidden. All phones and electronic devices must be turned off and kept in your bags: do not leave them on you. If you are seen to have an electronic device on your person, we may ask you to leave the exam immediately, and fraud allegations could be made, which could lead to a mark of 0 (zero) on this midterm.

Good luck!!!

By signing below, you acknowledge that you are required to respect the above statements.

Signature: _____

Do not write in the following table.

Question	1	2	3	4	5	6	Total
Maximum	5	3	4	3	2	3	20
Score							

1. (a) **[3 points]** Find the inverse of the following matrix, if it exists.

$$A = \begin{bmatrix} 3 & -1 & 3 \\ 0 & 2 & 1 \\ -1 & 2 & 0 \end{bmatrix}$$

- (b) **[2 points]** Consider the 3×3 matrix B with inverse $B^{-1} = \begin{bmatrix} 3 & -2 & -4 \\ 2 & -4 & 0 \\ -1 & 6 & 4 \end{bmatrix}$. Find

the solution to $B\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

2. [3 points] Indicate which of the following statements are (always) true or (possibly) false. For this question, no justification is required.

(a) If A is an $n \times n$ invertible matrix, then the columns of A are linearly independent.

Answer: _____

(b) If A is an $n \times n$ invertible matrix, then the homogeneous matrix equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

Answer: _____

(c) Suppose that A is an $n \times n$ invertible matrix. If B is an $n \times p$ matrix such that $AB = \mathbf{0}$, then $B = \mathbf{0}$, where $\mathbf{0}$ is the zero matrix.

Answer: _____

(d) If A is an $n \times n$ matrix such that $A^2 = \mathbf{0}$ (where $\mathbf{0}$ denotes the zero matrix) then A is invertible.

Answer: _____

(e) For A and B , two $n \times n$ invertible matrices, we have $(AB)^{-1} = B^{-1}A^{-1}$.

Answer: _____

(f) If, for an $n \times n$ matrix A , we have $A\mathbf{x} = \mathbf{b}$ is inconsistent for some vector $\mathbf{b} \in \mathbb{R}^n$, then A is invertible.

Answer: _____

3. Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 2 \end{bmatrix}$$

and let I be the 3×3 identity matrix. Compute the following.

(a) [**2 points**] $(AB^t)^{-1}$

(b) [**2 points**] $(A^tA) - 2I$

5. [2 points] Let A , B , C and X be $n \times n$ invertible matrices. Solve the following equation for the matrix X

$$C^{-1}(A + BXC)B^{-1} = I,$$

where I is the $n \times n$ identity matrix.

6. [3 points] Consider the following vectors of \mathbb{R}^4

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ -3 \\ -5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ -6 \\ 6 \\ 9 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 6 \\ h \\ -10 \end{bmatrix}$$

Determine the values of h , if any, for which $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent? Justify your answer.

Extra page