University of Ottawa

Department of Mathematics and Statistics

MAT 1302A : Mathematical Methods II
Professor: Hadi Salmasian

First Midterm Exam – Version A

February 2, 2018

Surname ______________________ First Name ______________________

Student # ______________________ DGD ______________________

Instructions:

(a) You have 80 minutes to complete this exam.
(b) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(c) Write your student number at the top of each page in the space provided.
(d) No notes, books, scrap paper, calculators or other electronic devices are allowed.
(e) You are strongly recommended to write in **pen**, not pencil.
(f) You may use the last page of the exam as scrap paper.
(g) Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. **Phones and devices must be turned off and put away in your bag**. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature ______________________

Please do not write in the table below.

Question	1	2	3	4	5	6	7	Total
Maximum	4	3	2	4	4	4	2	23
Grade								

1. [**4 points**] Write down the general solution of the following linear system in vector parametric form.

$$\left\{\begin{array}{rrrrl} x & -3y & +z & +2t & = 2 \\ & & z & +2t & = -1 \\ -x & +3y & -2z & -4t & = -1 \\ 2x & -6y & +4z & +8t & = 2 \end{array}\right.$$

2. [**3 points**] Determine all values of the parameter h such that the linear system

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & h & 2 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$

is consistent. You should justify your answer.

3. Compute the following:

(a) [**1 point**] $\begin{bmatrix} -1 & 3 \\ \frac{1}{2} & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix}$

(b) [**1 point**] $-3\mathbf{a} + 2A\mathbf{b}$ where $\mathbf{a} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$.

4. (a) [**2 points**] Let A be a 4×4 matrix, and set $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ -3 \\ -\frac{1}{3} \end{bmatrix}$, and $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

We know that the general solution of the linear system $A\mathbf{x} = \mathbf{b}$ is:

$$\begin{cases} x_1 & = -3x_2 + \frac{1}{3}x_4 - 3 \\ x_2 & : \text{free} \\ x_3 & = -2x_4 - \frac{1}{2} \\ x_4 & : \text{free} \end{cases}$$

Is the linear system $A\mathbf{x} = \mathbf{0}$ consistent? If your answer is "Yes", you should write down the general solution of the system $A\mathbf{x} = \mathbf{0}$.

(b) [**2 points**] Is it true that for any choice of b_1, b_2, b_3 in $\mathbb{R}$, the linear system

$$\begin{cases} x_1 & & -2x_3 & +3x_4 & -x_5 & = & b_1 \\ & -x_2 & +x_3 & & +5x_5 & = & b_2 \\ 2x_1 & -x_2 & -3x_3 & +6x_4 & +3x_5 & = & b_3 \end{cases}$$

is consistent? You should justify your answer.

5. [**4 points**] Set

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \ \mathbf{a}_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ -2 \end{bmatrix}, \ \mathbf{a}_3 = \begin{bmatrix} 1 \\ -2 \\ -3 \\ 1 \end{bmatrix}, \quad \text{and } \mathbf{b} = \begin{bmatrix} -3 \\ 0 \\ -3 \\ -7 \end{bmatrix}.$$

Does $\mathbf{b}$ belong to $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

6. [**4 points**] For each of the following statements, indicate if it is true or false. You will receive 1 point for every correct answer and lose .5 points for every incorrect answer (but you cannot receive a negative mark on this question).

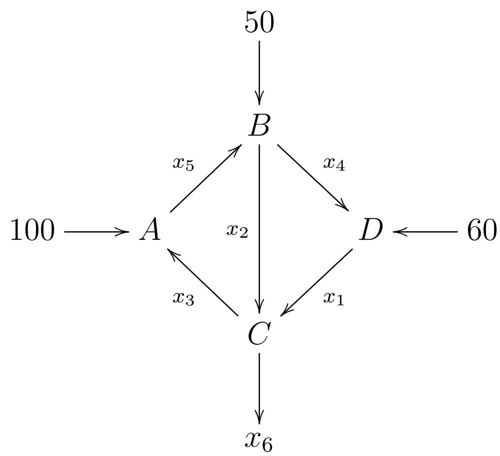
_____ Vectors of the form $\begin{bmatrix} 1 & -3 & 5 \\ 2 & -\frac{1}{2} & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, where $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is any vector in $\mathbb{R}^3$, are always in Span$\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$, where $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -3 \\ -\frac{1}{2} \\ 2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$.

_____ Two row equivalent matrices always have the same number of pivot columns.

_____ A homogeneous linear system always has infinitely many solutions.

_____ A linear system with 6 equations and 7 variables is always consistent.

7. [**2 points**] Consider the traffic flow described by the following diagram. The letters A through D label intersections. The arrows indicate the direction of flow (all roads are one-way) and their labels indicate flow in cars per hour.



Write down a linear system describing the traffic flow, i.e., all constraints on the variables $x_i, i = 1, \ldots, 6$. (You do not need to solve the linear system.)

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