

Part A: Answer Only Questions

For Questions 1–13, only your final answer will be considered for marks. Write your final answers in the spaces provided.

1. [2 points] Consider the matrices

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & -3 & -\frac{1}{3} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -4 \\ 3 & -3 \end{bmatrix}$$

Compute $B^T A$.

Final Answer: _____

2. [2 points] Suppose that A is a 4×7 matrix and B is a 6×4 matrix. For each statement below, write ‘**T**’ if the statement is true, and write ‘**F**’ if the statement is false. You will receive 0.5 points for each correct answer, lose 0.25 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

_____ The columns of B can never form a basis of \mathbb{R}^6 .

_____ $\text{Nul}(BA)$ is a subspace of \mathbb{R}^6 .

_____ The reduced echelon form of A always has at least one non-pivot column.

_____ $\text{rank}(BA) = 7 - \dim \text{Nul}(BA)$.

Student # _____

MAT 1302A Final Exam

3. [2 points] Suppose that A , B , and C are 4×4 matrices such that $\det(A) = -\frac{1}{2}$, $\det(B) = 5$, and $\det(-A^3BC^T) = 15$. Calculate $\det(C)$.

Final Answer: _____

4. [1 point] Determine all values of $t \in \mathbb{R}$ such that the columns of the matrix

$$\begin{bmatrix} 3 & -4 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 5 & t & -1 \\ 0 & -5 & 3 & \frac{1}{2} \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

are linearly dependent.

Final Answer: _____

5. [2 points] Let $z = 1 - 2i$ and $w = i + 2$. Write the complex number $\frac{w}{1 + \bar{z}}$ in the form $a + bi$ where a and b are real numbers.

Final Answer: _____

6. [2 points] Let $A = \begin{bmatrix} -3 & 0 & 0 \\ -2 & 1 & 0 \\ 3 - 2i & -1 + i & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Write down the eigenvalues of $(A + I)^2$ and their multiplicities.

Final Answer: _____

7. [1 point] Are the vectors

$$\begin{bmatrix} -3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 2 \\ 1 \end{bmatrix}$$

linearly independent? Write 'Yes' if these vectors are linearly independent, and write 'No' if they are linearly dependent.

Final Answer: _____

8. [2 points] Suppose we are given an $n \times n$ matrix A and a vector $\mathbf{b} \in \mathbb{R}^n$. Assume that the linear system $A\mathbf{x} = \mathbf{b}$ is inconsistent. For each statement below, write 'T' if it is true, and write 'F' if it is false. You will receive 0.5 points for each correct answer, lose 0.25 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

_____ Col A is not equal to \mathbb{R}^n .

_____ A is invertible.

_____ A has an eigenvector with eigenvalue $\lambda = 0$.

_____ $\mathbf{b} = \mathbf{0}_{n \times 1}$, where $\mathbf{0}_{n \times 1}$ denotes the zero vector in \mathbb{R}^n .

9. [2 points] Let \mathbf{a} , \mathbf{b} , and \mathbf{c} be vectors in \mathbb{R}^4 such that $2\mathbf{a} - 3\mathbf{b} = \mathbf{b} + 5\mathbf{c}$. Justify that \mathbf{a} belongs to $\text{Span}\{\mathbf{b}, \mathbf{c}\}$ by expressing \mathbf{a} explicitly as a linear combination of \mathbf{b} and \mathbf{c} .

Final Answer: _____

10. [1 point] Determine the value of the parameter t such that $\lambda = i$ is an eigenvalue of the matrix

$$A = \begin{bmatrix} 1 & t \\ -2 & 2i + 1 \end{bmatrix}.$$

Final Answer: _____

11. [2 points] For each of the following subsets of \mathbb{R}^3 , write 'Y' if the set is a *subspace* of \mathbb{R}^3 , and write 'N' if it is not. You will receive 0.5 points for each correct answer, lose 0.25 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

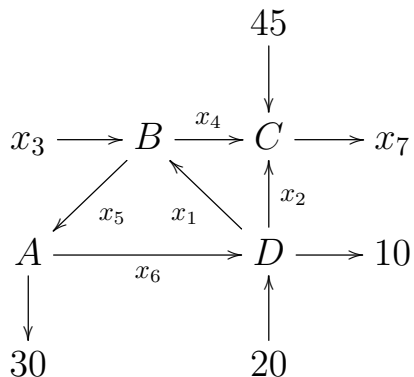
_____ $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid x_1 + x_2 + 1 = 0 \right\}$

_____ $\left\{ x \begin{bmatrix} -3 \\ 1 \\ -\frac{1}{2} \end{bmatrix} + y \begin{bmatrix} -2 \\ 0 \\ -7 \end{bmatrix} + z \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \mid x, y, z \in \mathbb{R} \right\}$

_____ The eigenspace of the matrix $A = \begin{bmatrix} 1 & -2 & \frac{3}{2} \\ -3 & 3 & 0 \\ 2 & 0 & 5 \end{bmatrix}$ corresponding to its eigenvalue $\lambda = 6$.

_____ $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x, y, z \in \mathbb{R} \text{ and } y = x^2 \right\}$

12. [2 points] Consider the traffic flow described by the following diagram. The letters A through D label intersections. The arrows indicate the direction of flow (all roads are one-way) and their labels indicate flow in cars per minute.



Write down a linear system describing the traffic flow, i.e., all constraints on the variables $x_i, i = 1, \dots, 7$. (Do not solve the linear system.)

Final Answer: _____

13. [2 points] Let A, B , and C be $n \times n$ invertible matrices. Solve the matrix equation $AX^TC - B = -BC$ for the matrix X .

Final Answer: _____

Part B: Long Answer Questions

For Questions 14–20, you must show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.

14. [4 points] Is the following linear system consistent or inconsistent? If it is consistent, then write down the general solution in vector parametric form.

$$\begin{cases} x_1 & -2x_3 & -x_4 & -2x_5 & = & 1 \\ & x_2 & +5x_3 & & +x_5 & = & -7 \\ 2x_1 & & -4x_3 & +x_4 & -7x_5 & = & 11 \end{cases}$$

Student # _____

MAT 1302A Final Exam

15. [4 points] Calculate the determinant of the following matrix using the method of co-factor expansion.

$$M = \begin{bmatrix} 0 & 15 & 0 & -1 \\ 5 & -3 & 7 & -2 \\ 0 & 3 & 0 & 0 \\ 1 & 11 & -1 & 9 \end{bmatrix}.$$

Student # _____

MAT 1302A Final Exam

16. (a) [4 points] Consider the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ -1 & -2 & 0 \\ -4 & 0 & 3 \end{bmatrix}$. Calculate the eigenvalues of A .

(b) [1 point] Is A diagonalizable? You should justify your answer.

Student # _____

MAT 1302A Final Exam

(c) **[3 points]** Let $B = \begin{bmatrix} 3 & 1 & 3 \\ -1 & 1 & -3 \\ 2 & 2 & 8 \end{bmatrix}$. Find a basis for the eigenspace of B corresponding to the eigenvalue $\lambda = 2$.

Student # _____

MAT 1302A Final Exam

17. The city of Quicheton has two culinary arts institutes: *ChefAcademie* and *RoyalToque*. In the beginning of January, the two institutes had equal student enrolment levels. During January, *ChefAcademie* updated its classes. As a result, at the end of January $\frac{1}{10}$ of *ChefAcademie*'s students switched to *RoyalToque*, while $\frac{1}{4}$ of *RoyalToque*'s students switched to *ChefAcademie*.

(a) [1 point] Write down the migration matrix M and the initial state vector \mathbf{x}_0 for this problem.

(b) [1 point] What fraction of the total number of students will be enrolled at each of the institutes in the beginning of February?

(c) [4 points] If the same migration trend continues for several months, then in the long run what are the predicted enrolment ratios of each of the institutes? In your final answer, you should clearly indicate the fraction of students for each institute.

Student # _____

MAT 1302A Final Exam

18. [4 points] Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & -3 \\ 2 & 1 & -5 & 7 & -5 \\ -1 & 2 & -10 & 14 & 4 \end{bmatrix}.$$

Find a basis for $\text{Nul } A$.

Student # _____

MAT 1302A Final Exam

19. [4 points] Let

$$A = \begin{bmatrix} 1 & -2 & -4 \\ 1 & -1 & -3 \\ 2 & 0 & -6 \end{bmatrix}$$

Find the inverse of A .

20. An economy consists of two sectors: Agriculture and Service. In order to produce one unit, the Agriculture sector consumes $\frac{1}{3}$ units from Agriculture and $\frac{1}{3}$ units from Service. Also, in order to produce one unit, the Service sector consumes $\frac{2}{3}$ units from Agriculture and $\frac{1}{6}$ units from Service.

- (a) [1 point] Write the consumption matrix for this economy.
- (b) [1 point] Write the Leontief Input-Output Model production equation.
- (c) [3 points] Determine the production levels needed to satisfy a final demand of 12 units from the Agriculture sector and 6 units from the Service sector.

Student # _____

MAT 1302A Final Exam

Extra page for answers.

Student # _____

MAT 1302A Final Exam

This page is intentionally left blank. You may use it as scrap paper.