

University of Ottawa
Department of Mathematics and Statistics

MAT 1302A : Mathematical Methods II
Professor: Hadi Salmasian

First Midterm Exam – Version A

February 5, 2016

Surname _____ First Name _____

Student # _____ DGD _____

Instructions:

- (a) You have 80 minutes to complete this exam.
- (b) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (c) Write your student number at the top of each page in the space provided.
- (d) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (e) You are strongly recommended to write in **pen**, not pencil.
- (f) You may use the last page of the exam as scrap paper.
- (g) Cellular phones, unauthorized electronic devices or course notes (unless an open-book exam) are not allowed during this exam. **Phones and devices must be turned off and put away in your bag**. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

Signature _____

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	4	4	3	3	2	3	19
Grade							

Student # _____

MAT 1302A First Midterm Exam

1. (a) **[3 points]** Find the general solution of the following linear equation. Write down the answer in vector parametric form.

$$\begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}.$$

(b) **[1 point]** Without using row reduction, find a solution of the vector equation

$$x_1 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

which satisfies $x_3 = -2$.

Hint: first use your solution for part (a) to write down the general solution for part (b).

Student # _____

MAT 1302A First Midterm Exam

2. [4 points] For each of the following statements, determine if it is **true** or **false**.

For each correct answer, you will receive 1 point. For each incorrect answer, you will lose 1 point, but your total score for this question cannot be negative.

_____ The number of pivot positions of the augmented matrix of a linear system is always equal to the number of its basic variables.

_____ Every vector in the span of the columns of a 4×3 matrix A is of the form $A\mathbf{x}$ for some vector \mathbf{x} in \mathbb{R}^3 .

_____ A homogeneous system can never be inconsistent.

_____ The matrix $\begin{bmatrix} \frac{1}{2} & 2 & \frac{2}{3} & -\sqrt{2} & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

Student # _____

MAT 1302A First Midterm Exam

3. Calculate the following.

(a) [1 point] $-\begin{bmatrix} 1 \\ \frac{1}{2} \\ 1 \end{bmatrix} + 3\begin{bmatrix} 0 \\ -\frac{1}{2} \\ 2 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix} =$

(b) [2 points] $\begin{bmatrix} 1 & -3 & \frac{1}{2} \\ 0 & 1 & -5 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} =$

Student # _____

MAT 1302A First Midterm Exam

4. **[3 points]** Determine all values of h and k such that the linear system

$$\begin{cases} x + y - kz = 1 \\ hx + 2y + 2z = -1 \end{cases}$$

is inconsistent.

Student # _____

MAT 1302A First Midterm Exam

5. [2 points] Suppose that \mathbf{u} , \mathbf{v} , \mathbf{w}_1 , \mathbf{w}_2 , and \mathbf{w}_3 are vectors in \mathbb{R}^m such that

$$\mathbf{u} = -\mathbf{w}_1 + \mathbf{w}_2 + 2\mathbf{w}_3 \quad \text{and} \quad \mathbf{v} = 2\mathbf{w}_1 + 3\mathbf{w}_2 - \mathbf{w}_3.$$

Express $\mathbf{u} + 2\mathbf{v}$ as a linear combination of \mathbf{w}_1 , \mathbf{w}_2 , and \mathbf{w}_3 . In other words, find coefficients $c_1, c_2, c_3 \in \mathbb{R}$ such that $\mathbf{u} + 2\mathbf{v} = c_1\mathbf{w}_1 + c_2\mathbf{w}_2 + c_3\mathbf{w}_3$.

6. [3 points] Consider the vectors

$$\mathbf{u} = \begin{bmatrix} -1 \\ -2 \\ \frac{1}{2} \\ -2 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -\frac{7}{2} \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 4 \\ -1 \\ -2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -\frac{1}{2} \\ -4 \end{bmatrix}$$

Determine if \mathbf{u} belongs to $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Student # _____

MAT 1302A First Midterm Exam

This page is intentionally left blank. You may use it as scrap paper.