

Student # _____

MAT 1302B Third Midterm Test – Blue Version

QUESTION 1. [5 points] Find a basis of the null space of the matrix

$$B = \begin{bmatrix} 1 & 1 & 2 & 0 & -1 & -2 \\ 0 & 1 & -2 & -1 & 3 & -5 \\ 0 & 0 & 0 & 1 & -1 & 3 \end{bmatrix}.$$

What is rank B ?

QUESTION 2. [4 points] For each of the following sets, write ‘Y’ if the set is a subspace of \mathbb{R}^n for the given n and write ‘N’ if the set is not a subspace of \mathbb{R}^n for the given n . You will receive 0.5 points for each correct answer, lose 0.5 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

- _____ The set of solutions to a homogeneous linear system in n variables. Here n can be any positive integer.
- _____ The set $\{(x - 3y, 2y - z, x + 2y + z) \mid x, y, z \in \mathbb{R}\}$, $n = 3$.
- _____ A line in \mathbb{R}^2 that does not pass through the origin, $n = 2$.
- _____ The span of 5 vectors in \mathbb{R}^4 , $n = 4$.
- _____ The set $\{(x, 1, x + y, y) \mid x, y \in \mathbb{R}\}$, $n = 4$.
- _____ The null space of a 3×5 matrix, $n = 5$.
- _____ The column space of a 7×8 matrix, $n = 8$.
- _____ A plane in \mathbb{R}^3 containing the origin, $n = 3$.

QUESTION 3. [4 points] For each statement below, indicate if it is true (T) or false (F). You will receive 0.5 points for each correct answer, lose 0.5 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

- _____ For every $n \times m$ matrix A , the equality $\text{rank } A + \dim \text{Nul } A = m$ holds.
- _____ If all of the entries of an $n \times n$ matrix A are integers, then the determinant of A is an integer.
- _____ The set $\{\vec{0}\}$ is a basis of the subspace $\{\vec{0}\}$.
- _____ Every nonconstant polynomial has a complex root.
- _____ The product of two imaginary numbers is an imaginary number.
- _____ If V is a subspace of \mathbb{R}^n , then every basis of V has n elements.
- _____ A subspace can have more than one basis.
- _____ A set of 3 vectors in \mathbb{R}^5 is always linearly independent.

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QUESTION 4. [4 points] Compute the determinant of the matrix

$$A = \begin{bmatrix} -3 & 0 & -2 & 0 & -1 \\ 10 & 4 & 17 & 25 & -18 \\ 5 & 0 & 3 & 0 & 2 \\ 3 & 0 & -10 & -1 & 8 \\ -1 & 0 & 2 & 0 & 0 \end{bmatrix}$$

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QUESTION 5. [3 points] Suppose $z = 2 - 3i$ and $w = -3 - 4i$. Calculate

$$\bar{z}w \quad \text{and} \quad \frac{z}{w}.$$

Write your answers in the form $a + bi$, where $a, b \in \mathbb{R}$.

QUESTION 6. Suppose A , B , and C are invertible 3×3 matrices with $\det A = 2$ and $\det C = -2$. Furthermore, suppose that

$$X = 2B^T C A^2 B^{-1} A^{-1}.$$

(a) [2 points] What is $\det X$?

(b) [1 point] Is X invertible?

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QUESTION 7. [5 points] Find a basis for

$$\text{Span} \left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} -4 \\ -4 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 2 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 10 \\ 16 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ -3 \\ -1 \\ 4 \\ 5 \end{bmatrix} \right\}$$

What is the dimension of this span?

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