

University of Ottawa  
Department of Mathematics and Statistics

MAT 1302B: Mathematical Methods II  
Professor: Alistair Savage

Second Midterm Test – Blue Version  
6 March 2015

Surname \_\_\_\_\_ First Name \_\_\_\_\_

Student # \_\_\_\_\_ DGD (1-4) \_\_\_\_\_

**Instructions:**

- (a) You have 80 minutes to complete this exam.
- (b) The number of points available for each question is indicated in square brackets.
- (c) Unless otherwise indicated, you must justify your answers to receive full marks.
- (d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (e) Write your student number at the top of each page in the space provided.
- (f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (g) You should write in *pen*, not pencil.
- (h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

| Question | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|----------|---|---|---|---|---|---|-------|
| Maximum  | 4 | 6 | 5 | 6 | 6 | 6 | 33    |
| Grade    |   |   |   |   |   |   |       |

QUESTION 1. [4 points] For each statement below, indicate if it is true (T) or false (F). You will receive 0.5 points for each correct answer, lose 0.5 points for each incorrect answer, and receive zero points for an answer left blank. You cannot receive a negative score on this question.

\_\_\_\_\_ For all  $n \times n$  matrices  $A$  and  $B$ , if  $A$  and  $B$  are invertible, then the product  $AB$  is invertible.

\_\_\_\_\_ For all  $n \times n$  matrices  $A$  and  $B$ , if  $A$  and  $B$  are invertible, then the sum  $A + B$  is invertible.

\_\_\_\_\_ For all  $n \times n$  matrices  $A$  and  $B$ , it is true that  $(AB)^T = A^T B^T$ .

\_\_\_\_\_ For all  $n \times n$  matrices  $A$  and  $B$ , it is true that  $(A + B)^T = A^T + B^T$ .

\_\_\_\_\_ The identity matrix  $I_6$  is invertible.

\_\_\_\_\_ If  $A$  is an  $n \times n$  invertible matrix, then there exists a vector  $\vec{b} \in \mathbb{R}^n$  such that the system  $A\vec{x} = \vec{b}$  is inconsistent.

\_\_\_\_\_ The matrix  $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$  is singular.

\_\_\_\_\_ If  $A$  is an  $n \times n$  matrix such that the equation  $A\vec{x} = \vec{0}$  has a nontrivial solution, then  $A$  is invertible.

## QUESTION 2.

(a) [2 points] Let

$$B = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Calculate  $B^2$ .

(b) [4 points] Let

$$A = \begin{bmatrix} 1 & -2 \\ 0 & -5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}.$$

Find the matrix  $X$  satisfying the equation

$$(3X^T + A)B = I_2.$$

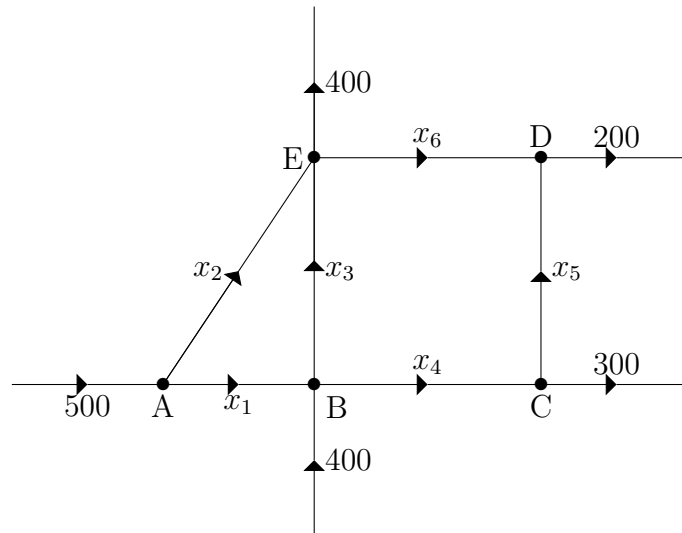
Here  $I_2$  is the  $2 \times 2$  identity matrix.

QUESTION 3. An economy consists of two sectors: industry and agriculture. In order to produce one unit, the industry sector consumes 0.4 units from industry and 0.8 units from agriculture. On the other hand, in order to produce one unit, the agriculture sector consumes 0.5 units from industry and 0.3 units from agriculture.

- (a) [**1 point**] Write the consumption matrix  $C$  for this economy.
- (b) [**1 point**] State the Leontief Input-Output Model production equation.
- (c) [**3 points**] Determine the production levels necessary to satisfy a final demand of 10 units from industry and 20 units from agriculture.

## QUESTION 4.

(a) [3 points] Write a system of linear equations describing the flow in the following network. The letters A through E label nodes in the network and the arrows indicate the direction of flow. You do *not* need to solve the system.



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(b) [**3 points**] The reduced echelon form of the augmented matrix associated to the linear system in part (a) is

$$\left[ \begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & 0 & -1 & 400 \\ 0 & 0 & 0 & 1 & 0 & 1 & 500 \\ 0 & 0 & 0 & 0 & 1 & 1 & 200 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

Find the general solution to this system. What are the minimum and maximum values for  $x_4$  and  $x_6$ ? (Use the fact that the flow along each edge in the network must be nonnegative.)

QUESTION 5. [6 points] Are the following sets linearly independent? Justify your answers.

(a)

$$\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

(b)

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

(c)

$$\left\{ \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \right\}$$

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QUESTION 6.

(a) [4 points] Is the matrix

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 9 \\ 0 & 1 & 2 \end{bmatrix}$$

invertible? If so, find its inverse.



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- (b) [**2 points**] Let  $A$  be the matrix from part (a). Solve the equation  $A\vec{x} = \vec{b}$ , where  $\vec{b}$  is the vector

$$\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

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