

QUESTION 1. [3 points] Which of the following sets are subspaces of \mathbb{R}^n for the given n ? You do not need to justify your answer.

- (a) $\{(2a, 3b - 4a, -2b + 1, 5c) \mid a, b, c \in \mathbb{R}\}$, $n = 4$.
- (b) $\{(2x, 0, -3y + x) \mid x, y \in \mathbb{R}\}$, $n = 3$.
- (c) The column space of a 6×3 matrix, $n = 6$.
- (d) The null space of a 5×4 matrix, $n = 5$.
- (e) The solution set of a homogeneous system in 4 variables with 6 equations, $n = 4$.
- (f) $\{(x, y) \mid 2x + y = 2\}$, $n = 2$.

Answer: _____

QUESTION 2. [3 points] Compute the following and write your answer in the form $a + bi$, $a, b \in \mathbb{R}$.

(a) $\begin{vmatrix} 2 + 3i & 4i \\ -3 & 3 - i \end{vmatrix}$

(b) $\frac{2 - i}{3 + 4i}$

QUESTION 3. [2 points] Which of the following statements are true? Note that more than one statement may be true. You should indicate *all* the true statements. (You will lose points for indicating that false statements are true, but you cannot receive a negative score on this question.)

- (a) The nullity of a matrix is the number of pivot columns of that matrix.
- (b) If A is an $m \times n$ matrix, then $\text{rank } A + \dim \text{Nul } A = n$.
- (c) An $n \times n$ matrix is invertible if and only if $\text{rank } A = n$.
- (d) A square matrix is invertible if and only if its transpose is invertible.
- (e) If A is a square matrix, then its column space has the same dimension as its null space.
- (f) The number $5 + 4i$ is an imaginary number.

Answer: _____

QUESTION 4.

(a) [2 points] Suppose

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 5.$$

What is

$$\begin{vmatrix} -4a_{11} & a_{13} & a_{12} \\ -4a_{21} & a_{23} & a_{22} \\ -4a_{31} & a_{33} & a_{32} \end{vmatrix} ?$$

Answer: _____

(b) [2 points] Suppose A , B , and C are 4×4 matrices with $\det A = -1$, $\det B = 3$, and $-2B^{-1}A^T C B^T = I$. What is the determinant of C ?

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QUESTION 5. [5 points] Compute the determinant of the matrix

$$A = \begin{bmatrix} 2 & 6 & 0 & -1 & 0 \\ 1 & 8 & 0 & 3 & 1 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 \\ 6 & -3 & -2 & 3 & 0 \end{bmatrix}.$$

QUESTION 6. [4 points] Consider the matrix

$$A = \begin{bmatrix} 1 & 6 & 2 & -5 & -2 \\ 0 & 0 & 2 & -8 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) Find a basis of $\text{Nul } A$.

(b) What is the dimension of $\text{Nul } A$?

QUESTION 7. [4 points] Consider the matrix

$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & -3 & -6 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}.$$

(a) Find a basis for $\text{Col } A$.

(b) What is the rank of A ?

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