

University of Ottawa
Department of Mathematics and Statistics

MAT 1302A: Mathematical Methods II
Instructor: Alistair Savage

First Midterm Test – White Version
31 January 2014

Surname _____ First Name _____

Student # _____ DGD (1-4) _____

Instructions:

- (a) You have 80 minutes to complete this exam.
- (b) The number of points available for each question is indicated in square brackets.
- (c) Unless otherwise indicated, you must justify your answers to receive full marks.
- (d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (e) Write your student number at the top of each page in the space provided.
- (f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (g) You should write in *pen*, not pencil.
- (h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	3	2	5	5	4	6	25
Grade							

QUESTION 1. Calculate the following.

(a) [1 point] $\begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 0 \\ 6 \end{bmatrix} =$

(b) [2 points] $\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} =$

QUESTION 2. [2 points] Which of the following statements are true? Note that more than one statement may be true. You should indicate *all* the true statements. (You will lose points for indicating that false statements are true, but you cannot receive a negative score on this question.)

- (a) A homogeneous systems always has infinitely many solutions.
- (b) If the coefficient matrix of a linear system is a 4×5 matrix with 4 pivot columns, then the system is consistent.
- (c) A matrix may have more than one reduced echelon form.
- (d) If one of the rows of an echelon form of an augmented matrix is of the form $[0 \ 0 \ 0 \ 1 \ 0]$, then the associated linear system must be consistent.
- (e) The span of two nonzero parallel vectors in \mathbb{R}^3 is a line.

Answer: _____

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QUESTION 3. [5 points] Find all the solutions to the following linear system.

$$\begin{array}{rccccrcr} & & x_2 & + & 2x_3 & = & 3 \\ x_1 & + & & & + & 3x_3 & = & 1 \\ 4x_1 & - & 3x_2 & + & 8x_3 & = & 1 \\ 2x_1 & - & 3x_2 & + & 2x_3 & = & -1 \end{array}$$

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QUESTION 4. [5 pts] For which values of h and k does the matrix-vector equation

$$\begin{bmatrix} 1 & 2h \\ 3 & 6 \end{bmatrix} \vec{x} = \begin{bmatrix} 2 \\ k \end{bmatrix}$$

- (a) have no solutions,
- (b) have a unique solution,
- (c) have infinitely many solutions?

QUESTION 5. [4 pts] Consider the vectors

$$\vec{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix} -1 \\ 4 \\ 17 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} c \\ 0 \\ d \end{bmatrix}.$$

For which values of c and d is the vector \vec{b} a linear combination of the vectors \vec{a}_1 , \vec{a}_2 , and \vec{a}_3 ?

QUESTION 6. [6 pts]

(a) Find the general solution to the following linear system:

$$\begin{array}{rclcl} x_1 & + & 2x_2 & - & 3x_3 & = & 0 \\ 2x_1 & + & x_2 & - & 3x_3 & = & 0 \\ -x_1 & + & x_2 & & & = & 0 \end{array}$$

Write your answer in *vector parametric form*. What is the geometric interpretation of the solution set?

(b) Check that

$$x_1 = 1, \quad x_2 = 2, \quad x_3 = 0$$

is a solution to the linear system

$$\begin{array}{rclcl} x_1 & + & 2x_2 & - & 3x_3 & = & 5 \\ 2x_1 & + & x_2 & - & 3x_3 & = & 4 \\ -x_1 & + & x_2 & & & = & 1 \end{array}$$

Without performing any row reduction, find the general solution to this linear system in *vector parametric form*. (Note that the coefficient matrix here is the same as in part (a).)

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