

QUESTION 1. [2 points] Which of the following subsets are subspaces of \mathbb{R}^n ? Note that more than one set may be a subspace. You should indicate *all* the subspaces. (You will lose points for incorrect answers, but you cannot receive a negative score on this question.)

- (a) The set of solutions to the matrix-vector equation $A\vec{x} = \vec{b}$, where A is an $m \times n$ matrix and \vec{b} is a nonzero vector in \mathbb{R}^m .
- (b) The set of solutions to the matrix-vector equation $A\vec{x} = \vec{0}$, where A is an $m \times n$ matrix.
- (c) The set $\{(2a - 3b, b + 7a, 8a + b) \mid a, b \in \mathbb{R}\}$. Here $n = 3$.
- (d) The set $\{(c + 5, c) \mid c \in \mathbb{R}\}$. Here $n = 2$.
- (e) The span of 5 vectors in \mathbb{R}^4 . Here $n = 4$.
- (f) The set $\{(x^2, x) \mid x \in \mathbb{R}\}$. Here $n = 2$.

Answer: _____

QUESTION 2. [2 points] Which of the following statements are false? Note that more than one statement may be false. You should indicate *all* the false statements. (You will lose points for indicating that true statements are false, but you cannot receive a negative score on this question.)

- (a) A square matrix A is invertible if and only if $\det A \neq 0$.
- (b) If $\vec{v}_1, \dots, \vec{v}_k$ are eigenvectors of a matrix A that correspond to distinct eigenvalues, then these vectors are linearly independent.
- (c) The eigenvalues of any square matrix are the entries on its diagonal.
- (d) For every matrix A , we have $\text{rank } A = \dim \text{Nul } A$.
- (e) For every square matrix A , we have $\det A = \det A^T$.
- (f) For any three vectors \vec{v}_1, \vec{v}_2 and \vec{v}_3 in \mathbb{R}^n , the subspace $\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ has dimension 3.

Answer: _____

QUESTION 3. Consider the matrix

$$M = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ -2 & 8 & 3 & 0 \\ 8 & 11 & -10 & 0 \end{bmatrix}.$$

(a) [**2 points**] What are the eigenvalues of M^2 ?

Answer: _____

(b) [**1 point**] What is $\det M^2$?

Answer: _____

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QUESTION 4. [2 points] Suppose A and B are 3×3 matrices with $\det A = 2$ and $\det B = -1$. Calculate $\det(-A^2B^T A)$.

QUESTION 5. [2 points] Consider the matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}.$$

Suppose that $\det A = 4$. What is $\det B$, where

$$B = \begin{bmatrix} a & b & c \\ d - 5a & e - 5b & f - 5c \\ 3g & 3h & 3i \end{bmatrix} ?$$

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QUESTION 6. [6 points] The eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

are 1 and 5. Find bases for the corresponding eigenspaces.

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Additional space for Question 6.

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QUESTION 7. [5 points] Compute the determinant of

$$A = \begin{bmatrix} 1 & 2 & 0 & 8 & 3 \\ 2 & 0 & 0 & -7 & -1 \\ -4 & 6 & 2 & 5 & 3 \\ 3 & -2 & 0 & 13 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

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QUESTION 8. Consider the matrix

$$A = \begin{bmatrix} -8 & 0 & -24 & -8 & -9 \\ 2 & 1 & 8 & 4 & 1 \\ 1 & 0 & 3 & 1 & 1 \\ -1 & 1 & -1 & 1 & 0 \end{bmatrix}.$$

The reduced row echelon form of A is

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) [**2 points**] Find a basis of $\text{Nul } A$.

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(b) [**1 point**] What is the dimension of $\text{Nul } A$?

(c) [**2 points**] Find a basis of $\text{Col } A$.

(d) [**1 point**] What is the rank of A ?

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