

University of Ottawa  
Department of Mathematics and Statistics

MAT 1302A: Mathematical Methods II  
Instructor: Alistair Savage

Second Midterm Test – White Version  
1 March 2013

Surname \_\_\_\_\_ First Name \_\_\_\_\_

Student # \_\_\_\_\_ DGD (1-4) \_\_\_\_\_

**Instructions:**

- (a) You have 80 minutes to complete this exam.
- (b) The number of points available for each question is indicated in square brackets.
- (c) Unless otherwise indicated, you must justify your answers to receive full marks.
- (d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (e) Write your student number at the top of each page in the space provided.
- (f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (g) You should write in *pen*, not pencil.
- (h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	2	5	4	6	4	5	26
Grade							

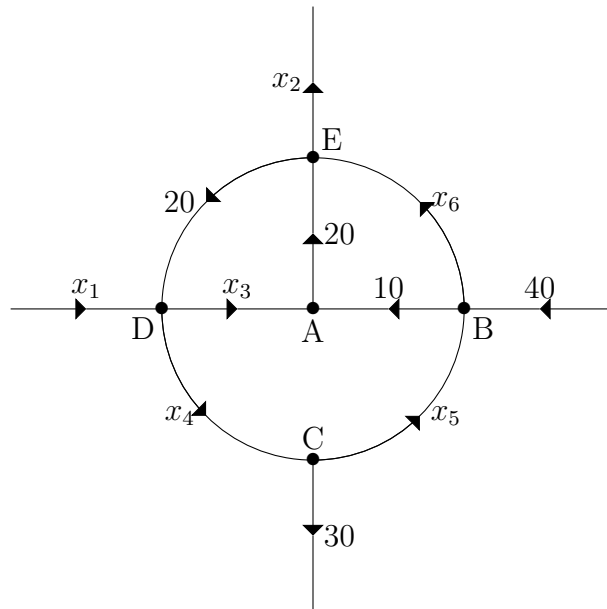
QUESTION 1. [2 points] Which of the following statements are true? Note that more than one statement may be true. You should indicate *all* the true statements. (You will lose points for indicating that false statements are true, but you cannot receive a negative score on this question.)

- (a) Every nonzero square matrix has an inverse.
- (b) If a matrix  $A$  is invertible, then  $A^T$  is invertible
- (c) If  $A$  and  $B$  are matrices such that  $AB = 0$ , then either  $A = 0$  or  $B = 0$ .
- (d) We can use the inverse matrix method to solve any matrix-vector equation  $A\vec{x} = \vec{b}$ .
- (e) If  $A$  and  $B$  commute, then  $(AB)^{-1} = A^{-1}B^{-1}$ .

Answer: \_\_\_\_\_

## QUESTION 2.

(a) [3 points] Write down a system of equations describing the flow in the following network. The letters A through E label intersections. The arrows indicate the direction of flow. Include all relevant equations. You do *not* need to solve the system.



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(b) [**2 points**] The reduced row echelon form of the augmented matrix for the linear system from part (a) is

$$\left[ \begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & -1 & -10 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -30 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

Write the general solution for the flow in the network. Indicate the ranges of possible values of any free variables (that is, give any minimum and maximum values of any free variables).

QUESTION 3. [4 pts] Are the following sets of vectors linearly independent? Justify your answers.

(a)

$$\left\{ \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ -2 \end{bmatrix} \right\}$$

(b)

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(c)

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$$

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QUESTION 4.

(a) [4 pts] Show that the following matrix is invertible and find its inverse:

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 8 & 6 \\ 4 & 14 & 7 \end{bmatrix}.$$

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(b) [2 pts] Let

$$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

Solve the system

$$A\vec{x} = \vec{b}.$$

(Here  $A$  is the same matrix as in part (a).)

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QUESTION 5. [4 points] Let  $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$ . Calculate  
 $A^{-1}(I_2 + B) + B^T$ .



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**QUESTION 6. [5 points]** A closed economy consists of two sectors: transportation and agriculture. Suppose that the consumption matrix of this economy is

$$C = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.8 \end{bmatrix}$$

where the first row and column correspond to transportation and the second row and column correspond to agriculture.

(a) Give the Leontief Input-Output Model production equation.

(b) Determine the production levels necessary to satisfy a final demand of  $\vec{d} = \begin{bmatrix} 30 \\ 20 \end{bmatrix}$ .

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