University of Ottawa
Department of Mathematics and Statistics

MAT 1302A: Mathematical Methods II
Instructor: Alistair Savage

First Midterm Test – White Version
1 February 2013

Surname _____ First Name _____

Student # _____ DGD (1–4) _____

Instructions:

(a) You have 80 minutes to complete this exam.
(b) The number of points available for each question is indicated in square brackets.
(c) Unless otherwise indicated, you must justify your answers to receive full marks.
(d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(e) Write your student number at the top of each page in the space provided.
(f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
(g) You should write in *pen*, not pencil.
(h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	2	3	6	5	4	5	25
Grade							

QUESTION 1. **[2 points]** Which of the following statements are true? Note that more than one statement may be true. You should indicate *all* the true statements. (You will lose points for indicating that false statements are true, but you cannot receive a negative score on this question.)

(a) A homogeneous linear system is always consistent.
(b) Every matrix is row equivalent to exactly one matrix in row echelon form.
(c) Every consistent linear system with 6 equations and 6 variables has exactly one solution.
(d) The span of *any* two vectors in $\mathbb{R}^3$ is a plane.
(e) A linear system is consistent if and only if its coefficient matrix and augmented matrix have the same number of pivot positions.

Answer:_____

QUESTION 2. Compute the following.

(a) **[1 point]** $\begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} - 4 \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix} =$

(b) **[2 points]** $\begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} =$

QUESTION 3. [**6 pts**] Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 3 \\ -3 & -7 & 1 \end{bmatrix}.$$

(a) Find all solutions to the homogeneous system $A\vec{x} = \vec{0}$ in *vector parametric form*. What is the geometric interpretation of the solution set?

(b) Suppose that $\vec{b} \in \mathbb{R}^3$ and that

$$\vec{x} = \begin{bmatrix} 2 \\ 1 \\ -4 \end{bmatrix}$$

is a solution to the equation $A\vec{x} = \vec{b}$. Find all solutions to the equation $A\vec{x} = \vec{b}$ in *vector parametric form*. (Note that the matrix A here is the same matrix as in part (a).)

QUESTION 4. **[5 pts]** Find all solutions to the following linear system:

$$\begin{array}{rcrcrcr} & & -x_2 & + & 3x_3 & = & 1 \\ x_1 & + & 4x_2 & + & 2x_3 & = & 10 \\ -x_1 & - & 5x_2 & + & x_3 & = & -9 \\ x_1 & + & 4x_2 & + & 3x_3 & = & 11 \end{array}$$

QUESTION 5. [**4 pts**] Let

$$A = \begin{bmatrix} 1 & -3 & -4 \\ -4 & 2 & 8 \\ 3 & 11 & 4 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

For which values of b_1, b_2 and b_3 does the matrix-vector equation $A\vec{x} = \vec{b}$ have at least one solution? Justify your answer.

QUESTION 6. **[5 pts]** For which values of a and b does the linear system

$$\begin{array}{rcrcl} 2x_1 & + & 4x_2 & = & b \\ x_1 & + & ax_2 & = & 3 \end{array}$$

(a) have no solutions,
(b) have a unique solution,
(c) have infinitely many solutions?

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