

University of Ottawa
Department of Mathematics and Statistics

MAT 1302B: Mathematical Methods II
Instructor: Alistair Savage

Second Midterm Test – White Version
23 March 2012

Surname _____ First Name _____

Student # _____ DGD (1-4) _____

Instructions:

- (a) You have 80 minutes to complete this exam.
- (b) The number of points available for each question is indicated in square brackets.
- (c) Unless otherwise indicated, you must justify your answers to receive full marks.
- (d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (e) Write your student number at the top of each page in the space provided.
- (f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (g) You should write in *pen*, not pencil.
- (h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	5	5	6	3	5	4	28
Grade							

QUESTION 1. [5 pts]

(a) Calculate the determinant of the following matrix:

$$A = \begin{bmatrix} 2 & 6 & 6 & 0 \\ 0 & 2 & 3 & -1 \\ 1 & 5 & 0 & 1 \\ 3 & 7 & 0 & 7 \end{bmatrix}$$

(b) Is A invertible? Justify your answer. If A is invertible, find the determinant of A^{-1} .

QUESTION 2. [5 pts]

- (a) If possible, find the inverse of the matrix A below.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

- (b) Suppose $\vec{b} \in \mathbb{R}^3$. How many solutions does the equation $A\vec{x} = \vec{b}$ have? Justify your answer.

QUESTION 3. [6 pts] *The two parts of this question are independent of one another.*

(a) Suppose $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ -\frac{1}{2} & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -\frac{1}{3} & 5 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$ and C is a 3×3 matrix such that

$$A^{-1}B(C^T)^{-1}A^2B^{-2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

Find $\det C$. *Hint:* Use the properties of determinants.

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- (b) Suppose A and B are 4×4 invertible matrices. Find a matrix X satisfying the matrix equation

$$A^{-1} (I + 2B(3X + I)^T) A = 2I + A.$$

Simplify your answer as much as possible.

QUESTION 4. [3 pts] For each of the following subsets, state whether or not it is a subspace of \mathbb{R}^n for the given n . Justify your answers.

$$(a) H = \left\{ \begin{bmatrix} a \\ 1/a \\ b \end{bmatrix} \mid a, b \in \mathbb{R} \text{ and } a \neq 0 \right\}, \quad n = 3.$$

$$(b) W = \left\{ \begin{bmatrix} a \\ b \\ a + 2b \\ 0 \end{bmatrix} \mid a, b \in \mathbb{R} \right\}, \quad n = 4.$$

$$(c) V = \left\{ \begin{bmatrix} x \\ y \\ xy \end{bmatrix} \mid x, y \in \mathbb{R} \right\}, \quad n = 3.$$

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QUESTION 5. [5 pts] Consider the matrix

$$A = \begin{bmatrix} 0 & 1 & -1 & 0 & 2 \\ 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 2 \end{bmatrix}.$$

(a) Find a basis for $\text{Nul } A$.

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- (b) Find a basis for $\text{Col } A$. What is the rank of A ? Here A is the same matrix as in the previous part of this question.

QUESTION 6. [4 pts] Consider an economy divided into 2 sectors: Services and Transportation. In order to produce one unit of output, Services must consume 0.6 units from its own sector and 0.2 units from Transportation. On the other hand, to produce one unit of output, Transportation must consume 0.1 units from its own sector and 0.3 units from Services.

(a) Give the consumption matrix C for this economy.

(b) Find the intermediate demand if Services wants to produce 15 units and Transportation wants to produce 20 units.

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- (c) Determine the production levels needed to meet a final demand of 24 units from Services and 12 units from Transportation.

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