MAT 1302 D – Test 3 – March 29th ,Winter 2011

Instructor: Termeh Kousha

[Print your FAMILY NAME in CAPITAL letters]

Name:_____

Student Number:_____

Signature:_____

Make sure your cell phone is off before starting...

Instructions: This exam consists of 5 questions in 8 pages. The marks for each question are as listed with the question itself. The exam is out of **30**. No calculators or other electronic aids allowed. No notes, books or other papers allowed.
Write all your answers in **non-erasable pen**. If you make a mistake just scratch it out and continue. You may use the back of pages for answer of questions.

GOOD LUCK!

1. [3 points] Determine if the vectors are linearly independent. Justify your answer.

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}.$$

2. [3 points] **Just by inspection**, determine whether the vectors are linearly dependent. Justify your answer clearly.

a. $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix}$.

b. $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

c. $\begin{bmatrix} -4 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -8 \\ 2 \end{bmatrix}$.

3. [4 points] Find the determinant of $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & 2 \\ 1 & -1 & 3 & -3 \\ 2 & 1 & 1 & 0 \end{bmatrix}$.

For question 4, you have to show ALL the details. If you claim something, you need to prove it.

4.(a) [4 points] Show that if A and B are similar, then they have the same eigenvalues.

(b) [4 points] Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}.

5. [12 points] Let $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

a. Find the characteristic polynomial of A. List the eigenvalues and their multiplicities.

b. Find the all the corresponding eigenvectors.

c. If possible, diagonalize the matrix A. That is, find a diagonal matrix D and invertible matrix P such that $A = PDP^{-1}$. (Note: You are not asked to find P^{-1}.)

Extra page