BlUE

MAT 1302 D – .Test 2 – March 1st ,Winter 2011

Instructor: Termeh Kousha

[Print your FAMILY NAME in CAPITAL letters]

Name: Marking Scheme and Solutions.

Student Number:______________________________

Signature:___________________________________

Make sure your cell phone is off before starting...

Instructions: This exam consists of 5 questions in 7 pages. The marks for each question are as listed with the question itself. The exam is out of **30**, but there are **33** points possible. (3 bonus points)
No calculators or other electronic aids allowed. No notes, books or other papers allowed.
Write all your answers in **non-erasable pen**. If you make a mistake just scratch it out and continue. You may use the back of pages for answer of questions.

GOOD LUCK!

1 point for True / False

2 points " prove / counterexample

1. Mark each statement True or False. If it's True prove it, if it's false give a counterexample. Note that you can NOT prove that a statement is true by giving examples.

- (a) [3 points] If A, B are two $m \times n$ matrices, then both AB^T and $A^T B$ are defined.

True. $A_{m\times n} \rightarrow A^T_{n\times m}$, $B_{m\times n} \rightarrow B^T_{n\times m}$ so $A_{m\times n} B^T_{n\times m}$ is defined. $A^T_{n\times m} B_{m\times n}$ is " .

- (b) [3 points] If $A_{n\times n}$ is an invertible matrix, then the equation $Ax = \overrightarrow{b}$ is always consistent for ANY $\overrightarrow{b} \in \mathbb{R}^n$.

True ; A invertible $\rightarrow$ $A_{n\times n}$ is row equivalent To $I_{n\times n}$ (or A has n pivots)

(or $A\vec{x} = \vec{b} \Rightarrow \vec{x} = A^{-1}\vec{b}$] $\forall b$ $\rightarrow$ the system is always consistent for any $b \in \mathbb{R}^n$.

- (c) [3 points] If A and B are two $n \times n$ matrices, then if $AB = 0$ then either $A = 0$ or $B = 0$.

False, (any other counterexample is fine.)

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ $\Rightarrow AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A \neq 0$ $B \neq 0$

- (d) [3 points] If A and B are two $n \times n$ matrices, such that AB and B are both invertible, then A is also invertible.

True. let $AB = C \Rightarrow$ then C is invertible.
B: also invertible B^{-1} exists.

$A = (AB)B^{-1} = CB^{-1}$ $\rightarrow$ A is product of two invertible matrices $\Rightarrow$ A is also invertible

or / $AB = C \Rightarrow A = CB^{-1}$

- (e) [3 points] Every equation that has a solution in $\mathbb{C}$, has also a solution in $\mathbb{R}$.

False $x^2 + 1 = 0$ has no solution in $\mathbb{R}$ But has solution in $\mathbb{C}$

2. [4 points] Let $A = \begin{bmatrix} 0 & -1 & 0 \\ 2 & 2 & -3 \\ -1 & -1 & 1 \end{bmatrix}$. Find the inverse of A if it exists.

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 0 & 1 & 0 & 0 \\ 2 & 2 & -3 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right]$$

1

$$\overset{R_1 \leftrightarrow R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & 0 & -1 \\ 2 & 2 & -3 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{array}\right]$$

1

$$\overset{-2R_1+R_2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 1 & +2 \\ 0 & -1 & 0 & 1 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & 0 & -1 \\ 0 & +1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -2 \end{array}\right]$$

2

$$\underset{\substack{+R_3+R_1 \\ -R_2+R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & -3 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & -2 \end{array}\right] \Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & 0 & 0 \\ 0 & -1 & -2 \end{bmatrix}$$

I_n A^{-1}

A is invertible

NO points for checking.

check: $\left(\begin{bmatrix} 0 & -1 & 0 \\ 2 & 2 & -3 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -3 \\ -1 & 0 & 0 \\ 0 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$

3. [5 points] Let $A = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 0 & 0 \\ 2 & 2 \end{bmatrix}$. Find AB, $\frac{1}{2}(B^T A)$, $A^T B$ and $A + B$ when it's defined.

$A_{3\times 3}$ $B_{3\times 2}$

1 $$\begin{bmatrix} 2 & 1 & -2 \\ 0 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 0 & 0 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -8 \\ 4 & 4 \\ 0 & -4 \end{bmatrix}$$

2 $$B^T = \begin{bmatrix} 2 & 0 & 2 \\ -2 & 0 & 2 \end{bmatrix} \to B^T A = \begin{bmatrix} 6 & 6 & -6 \\ -2 & 2 & 2 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2} B^T A = \begin{bmatrix} 3 & 3 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

1 Note $(B^T A)^T = A^T B$

So $A^T B = \begin{bmatrix} 6 & -2 \\ 6 & 2 \\ -6 & 2 \end{bmatrix}$

(it's okey if they recalculate ATB).

1 $A + B$ Not defined.

4. [3 points] Consider the production model $x = Cx + d$ for an economy with two sectors, where

$$C = \begin{bmatrix} 0 & .5 \\ .6 & 0.2 \end{bmatrix} \quad d = \begin{bmatrix} 30 \\ 40 \end{bmatrix}.$$

Use an inverse matrix to determine the production level necessary to satisfy the final demand.

$$(I - C)\vec{x} = \vec{d}$$

$$\rightarrow \quad \vec{x} = (I - C)^{-1}\vec{d}$$

$$I - C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \\ 0.6 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -0.5 \\ -0.6 & +0.8 \end{bmatrix}$$

$$\det|I - C| = 0.8 - 0.3 = 0.5$$

$$(I - C)^{-1} = \frac{1}{\frac{1}{2}} \begin{bmatrix} 0.8 & 0.5 \\ 0.6 & 1 \end{bmatrix}$$

2

$$= \begin{bmatrix} 1.6 & 1 \\ 1.2 & 2 \end{bmatrix}$$

2 points

1 point

$$\vec{x} = \begin{bmatrix} 1.6 & 1 \\ 1.2 & 2 \end{bmatrix} \begin{bmatrix} 30 \\ 40 \end{bmatrix} = \begin{bmatrix} 48 + 40 \\ 36 + 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 116 \end{bmatrix}$$

or $[\, I - C : \vec{d} \,] \rightarrow$ find $\vec{x}$

5. [6 points] Let $z = 3-2i$, $w = -3-4i$ and $r = -5$. Calculate the following: (Write them in the form of $a + ib$)

1/ **a.** $z.w$

$(3-2i)\cdot(-3-4i) = -17-6i$

1/ **b.** $\frac{z}{w}$ $= \frac{3+2i}{-3+4i} \cdot \frac{-3-4i}{-3-4i} = \frac{-1-18i}{25} = \frac{-1}{25} - \frac{18}{25}i$

1/ **c.** $|rw|$

$= |r||w| = 5\sqrt{9+16} = 5 \cdot 5 = 25$.

2/ **d.** $\frac{1}{z} + \frac{1}{w}$

$= \frac{1}{3-2i} + \frac{1}{-3-4i} = \frac{1}{3-2i} \cdot \frac{3+2i}{3+2i} + \frac{1}{-3-4i} \cdot \frac{-3+4i}{-3+4i} =$

$\frac{3+2i}{13} + \frac{-3+4i}{25} = \frac{36}{325} + \frac{102}{325}i$

$=$

Extra Page