MAT 1302 D – Test 1 – Feb 1st ,Winter 2011

Instructor: Termeh Kousha

[Print your FAMILY NAME in CAPITAL letters]

Name: Solution and Marking Scheme   Orange

Student Number:____________________

Signature:____________________

Make sure your cell phone is off before starting...

Instructions: This exam consists of 6 questions in 7 pages. The marks for each question are as listed with the question itself. The exam is over **30**.
No calculators or other electronic aids allowed. No notes, books or other papers allowed.
Write all your answers in **non-erasable pen**. If you make a mistake just scratch it out and continue. You may use the back of pages for answer of questions.

GOOD LUCK!

1. Compute the following

(a) [1 mark] $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (-2)\begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 5 \end{bmatrix}$

(b) [2 mark] $\underset{2\times 3}{\begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & 2 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}} = \underset{2\times 1}{\begin{bmatrix} -1\times 1 - 1 + 2\times 1 \\ 0\times 1 + 3\times(-1) + 2\times 1 \end{bmatrix}}$

$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

2. [3 marks] Let $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$. Which one is in span$\left\{\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}\right\}$? Why?

$2\vec{u} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \Rightarrow$ so $\vec{u} \in \text{Span}\left\{\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}\right\}$

$\vec{v} \in \text{Span}\left\{\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}\right\}$ since $0\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} = \vec{0}$

$\vec{w} \notin \text{span}\left\{\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}\right\}$

$\nexists\, K$ s.t $K\vec{w} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$

3. [6 marks] For which values of h and k does the system

$$\begin{aligned} x_1 + 3x_2 &= k \\ 4x_1 + hx_2 &= 8 \end{aligned}$$

a. have no solution,

b. have a unique solution

c. have many solution.

$$\begin{bmatrix} 1 & 3 & K \\ 4 & h & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & K \\ 0 & -12+h & -4K+8 \end{bmatrix}$$

a. have no solution:

2/

$-12+h=0$ and $-4K+8 \neq 0$

$h=12$ $K \neq 2$

b. have a unique solution.

2/

$-12+h \neq 0$

$h \neq 12$

$K \in \mathbb{R}$

c. have many solution:

2/

$-12+h=0$ $h=12$ and

$-4K+8=0 \Rightarrow K=2$

4. [5 marks] Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 2 \\ -3 & -1 & 0 \end{bmatrix}$ and $\overrightarrow{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ be a matrix and a vector. For which $b_1,\ b_2,\ b_3$ is the matrix equation $A\overrightarrow{x} = \overrightarrow{b}$ consistent?

$$\begin{bmatrix} 1 & 3 & 4 & b_1 \\ -1 & 1 & 2 & b_2 \\ -3 & -1 & 0 & b_3 \end{bmatrix} \underset{\substack{R_1+R_2 \\ 3R_1+R_3}}{\sim} \begin{bmatrix} 1 & 3 & 4 & b_1 \\ 0 & 4 & 6 & b_1+b_2 \\ 0 & 8 & 12 & 3b_1+b_3 \end{bmatrix}$$

1/

$$\underset{-2R_2+R_3}{\sim} \begin{bmatrix} 1 & 3 & 4 & b_1 \\ 0 & 4 & 6 & b_1+b_2 \\ 0 & 0 & 0 & -2(b_1+b_2)+3b_1+b_3 \end{bmatrix}$$

2/

$$-2b_1 - 2b_2 + 3b_1 + b_3 = 0$$

$$\boxed{b_1 - 2b_2 + b_3 = 0} \rightarrow$$ the solution is a plane through the origin.

1

1

5. [5 marks] Suppose $\vec{a_1} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{a_2} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ -2 \end{bmatrix}$, $\vec{a_3} = \begin{bmatrix} -1 \\ 4 \\ 7 \\ 4 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 1 \end{bmatrix}$. Is $\vec{b}$ in the Span$\{\vec{a_1}, \vec{a_2}, \vec{a_3}\}$?

$$\begin{bmatrix} 0 & 3 & -1 & 2 \\ 1 & -2 & 4 & 1 \\ 2 & -1 & 7 & 3 \\ 1 & -2 & 4 & 1 \end{bmatrix} \underset{R_1 \leftrightarrow R_2}{\sim} \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 3 & -1 & 2 \\ 2 & -1 & 7 & 3 \\ 1 & -2 & 4 & 1 \end{bmatrix}$$

$$\underset{\substack{-R_1+R_4 \\ -2R_1+R_3}}{\sim} \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 3 & -1 & 2 \\ 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 $\quad \overset{-R_2+R_3}{\sim} \begin{bmatrix} 1 & -2 & 4 & 1 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

the system is inconsistent

2 $\quad \Rightarrow \vec{b} \notin \text{span}\{a_1, a_2, a_3\}$

6. [8 marks] Let $A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & -3 & -6 \end{bmatrix}$.

(a) Find the solution set of the homogeneous system $A\vec{x} = 0$ in parametric vector form. What is the geometric description?

(b) Describe the solution of $A\vec{x} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$ in the parametric vector form. Provide a geometric solution comparison with the solution set in part (a).

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ -4 & -9 & 2 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix} \underset{4R_1+R_2}{\sim} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & -3 & -6 & 0 \end{bmatrix}$$

$$\underset{R_2+R_3}{\sim} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 3 & 6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \underset{R_2/3}{\sim} \begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1 point

$$\underbrace{\underbrace{x_1 \quad x_2}_{\text{Basic}} \qquad \underset{\text{free}}{x_3}}_{\text{1 point}}$$

1 point

$x_1 - 5x_3 = 0$

$x_2 + 2x_3 = 0$

$x_3 = t$ free $t \in \mathbb{R}$

$$\begin{bmatrix} 5t \\ -2t \\ t \end{bmatrix} = t \underbrace{\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}}_{\vec{u}} = \text{Span}\{\vec{u}\}$$

Line passes through the origin

1 point

Extra Page

b)

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{bmatrix} \overset{RR}{\sim} \begin{bmatrix} 1 & 0 & -5 & -2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \text{Basic} & & \text{free.} \end{array}$$

1 point

$x_1 - 5x_3 = -2$ $\qquad$ $x_1 = -2 + 5t$

$x_2 + 2x_3 = 1$ $\qquad$ $x_2 = 1 - 2t$ $\qquad$ 1 point

$x_3 = t$ free $\qquad$ $x_3 = t$

$$\begin{bmatrix} -2+5t \\ 1-2t \\ t \end{bmatrix} = t\underbrace{\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}}_{\vec{u}} + \underbrace{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}}_{\vec{p}}$$

1 point

The solution set is the line through $\vec{p}$ and parallel to the line $t\vec{u}$ which is the set of Solutions of homogeneous system.

1 point.