

University of Ottawa  
Department of Mathematics and Statistics

MAT 1302B: Mathematical Methods II  
Professor: Alistair Savage

Second Midterm Test  
March 19, 2010

Surname \_\_\_\_\_ First Name \_\_\_\_\_

Student # \_\_\_\_\_ DGD (1-4) \_\_\_\_\_

**Instructions:**

- (a) You have 80 minutes to complete this exam.
- (b) The number of points available for each question is indicated in square brackets.
- (c) Unless otherwise indicated, you must justify your answers to receive full marks.
- (d) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- (e) Write your student number at the top of each page in the space provided.
- (f) No notes, books, scrap paper, calculators or other electronic devices are allowed.
- (g) You should write in **pen**, not pencil
- (h) You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	6	4	5	4	4	5	28
Grade							

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

1.

(a) **[3 pts]** Let

$$A = \begin{bmatrix} 3 & 2 & -1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & -1 & 3 & 3 \\ 2 & 0 & 1 & 0 \end{bmatrix}.$$

Compute the determinant of  $A$ .

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

(b) [1 pt] Find  $\det B$ , where

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & 0 & 0 \\ 4 & -1 & 4 & -1 & 0 \\ 1 & 2 & 0 & 5 & -1 \end{bmatrix}.$$

(c) [2 pts] Suppose  $C$  is a  $5 \times 5$  invertible matrix. Do you have enough information to compute  $\det(2C^{-1}B^2C^T)$ , where  $B$  is the matrix from Part (b)? If so, what is this determinant?

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

2. [4 pts] Suppose

$$M = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & -1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}.$$

Is  $M$  invertible? If so, find  $M^{-1}$ .

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

3. An economy has two sectors: Agriculture and Manufacturing. In order to produce one unit of output, Agriculture requires 0.7 units from its own sector and 0.1 units from Manufacturing. On the other hand, Manufacturing requires 0.5 units from its own sector and 0.5 units from Agriculture to produce one unit of output.

(a) [1 pt] Write down the consumption matrix  $C$  for this economy.

(b) [1 pt] Determine the intermediate demands if Agriculture produces 100 units of output.

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

- (c) [**3 pts**] Determine the production levels required to meet a final demand of 100 units from Agriculture and 150 units from Manufacturing.

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

4. Let

$$A = \begin{bmatrix} 1 & 0 & 1 & -1 & 3 & 1 & 0 \\ 1 & 0 & 3 & 0 & 8 & 8 & 0 \\ 1 & 0 & -1 & -2 & -2 & -3 & 0 \end{bmatrix}.$$

(a) [**2 pts**] Find a basis for  $\text{Col } A$ .

(b) [**1 pt**] What is  $\text{rank } A$ ?

(c) [**1 pt**] What is the nullity of  $A$ ? Remember that the nullity of  $A$  is  $\dim \text{Nul } A$ .

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

5. Let

$$B = \begin{bmatrix} 1 & 0 & 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(a) [**3 pts**] Find a basis for  $\text{Nul } B$ .

(b) [**1 pts**] What is  $\text{rank } B$ ?



Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

6. [5 pts] Which of the following sets are subspaces of  $\mathbb{R}^n$  for the given value of  $n$ ? Remember to justify your answer.

(a)  $A = \{(x, -x) \mid x \geq 0\}$ ,  $n = 2$ .

(b)  $B = \left\{ \left[ \begin{array}{c} 3x + 2y - z \\ -x + y + z \\ 2x \\ -5y \\ 4 \end{array} \right] \mid x, y, z \in \mathbb{R} \right\}$ ,  $n = 5$ .

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

$$(c) C = \left\{ \left[ \begin{array}{c} 3x + 2y - z \\ -x + y + z \\ 2x \\ -5y \\ z \end{array} \right] \mid x, y, z \in \mathbb{R} \right\}, n = 5.$$

Student # \_\_\_\_\_

MAT 1302B Second Midterm Test

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