1. Find all k for which the following system has infinitely many solutions.

$$\begin{aligned} x + 2y + z &= 0 \\ x + 3y + 2kz &= 0 \\ 2x + 3y + kz &= 0 \end{aligned}$$

A. 0
B. 1/2
C. 3
D. -3
E. 1
F. -1

2. The coefficient matrix A in a homogeneous system of 12 equations in 16 unknowns is known to have rank 6. How many parameters are there in the general solution?

A. 16
B. none
C. 10
D. 4
E. 12
F. 6

3. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$. What is the second row of A^{-1} ?

A. [13/8, -1/2, -1/8]
B. [-15/8, 1/2, 3/8]
C. [-15/8, 1/8, 3/8]
D. [-15, 4, 3]
E. [-1/2, 1/2, 0]
F. [-15/4, 1, 3/4]

4. Let A and B be 4×4 matrices with det A = 3 and det B = -5. Find $\det(2AB^tB^{-2})$.

A. 48/3
B. -36/5
C. 24/5
D. -48/5
E. 12/5
F. -24/5

5. Let A be an $n \times n$ matrix. Among the following statements, one is not equivalent to the other four. Which one is it?

A. The rank of A is n.
B. A is not invertible.
C. The equation Ax = b has a unique solution x for any n-vector b.
D. A can be row-reduced to the identity matrix I_n.
E. The columns of A are linearly independent.

6. Which two of the following are not subspaces of $\mathbf{R}^4$?

R = {(a, b, c, d) ½c = a + 2b, d = a - 3b}
S = {(a, b, c, d) ½a = 0, b = 0}
T = {(a, b, c, d) ½a - b = 2, c = d}
U = {(a, b, c, d) ½a ≥ 0, b ≥ 0}

A. T and U
B. S and U
C. R and T
D. R and U
E. S and T
F. T and U

7. What is the dimension of the subspace spanned by S = { (1, 1, 1), (-1, 1, -1), (1, 1, 3), (0, 2, 1) } ?

A. 0
B. 3
C. 1
D. 2
E. 4
F. These vectors do not span a subspace

8. A basis for the eigenspace corresponding to the eigenvalue 1 of the matrix $\begin{bmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ is:

A. { (-1, 4, 1) }
B. { (-1, 4, 1), (1, -1, -1) }
C. { (1, 2, 1) }
D. { (-2, 1, 3) }
E. { (1, 2, 1), (-1, 4, 1) }
F. { (1, -1, -1) }

9. Consider the data points $(1, 8), (3, 26), (5, 60)$.

a) Find a polynomial of degree two whose graph passes through the given points.

b) Using your result from (a), predict the value of y when $x = 2$.

c) How many polynomials of degree 3 will pass through the given points? You do not need to find them, but you must explain your answer.

10. Consider an economy consisting of two industries with input-output matrix given below.

$$A = \begin{bmatrix} 1/10 & 2/5 \\ 3/10 & 1/5 \end{bmatrix}$$

a) What is the amount of production available to the open sector if the total output in the industries are given by $X = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$?

b) What is the production level necessary to meet the open sector demand given by $D = \begin{bmatrix} 18 \\ 6 \end{bmatrix}$?

c) What is the meaning of the $(2, 1)$ entry of the matrix A above?

11. The following table gives occupational transistion probabilities.

	Initial generation			
	white-collar	manual		
	1	.2	white-collar	**Next generation**
	0	.8	manual	

a) Give the matrix of transition probabilities.

b) If the mother is a manual worker, what is the probability that her daughter or son will be a white-collar worker?

c) If there are 10,000 in the white-collar category and 20,000 in the manual category, what will the distribution be one generation later?

d) If the father is a manual worker, what is the probability that his grandaughter will be a manual worker?

e) What is the long-term distribution likely to be?

12. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{bmatrix}$.

a) Find the characteristic polynomial of A.

b) Show that the eigenvalues of A are 1 and 3.

c) Find a basis for the eigenspace E_1 corresponding to $\lambda = 1$. What is its dimension? Give a geometric description of E_1.

d) Find a basis for the eigenspace E_3 corresponding to $\lambda = 3$. What is its dimension? Give a geometric description of E_3.

13. Consider the homogeneous system of equations

$$\begin{array}{ccccccccc} 2x_1 & - & 4x_2+ & + & 12x_3 & - & 10x_4 & = & 0 \\ -1x_1 & + & 2x_2 & - & 3x_3 & + & 2x_4 & = & 0 \\ 2x_1 & - & 4x_2 & + & 9x_3 & - & 6x_4 & = & 0 \end{array}$$

a) Write down the augmented matrix and perform the first TWO steps in the Gaussian algorithm. DO NOT COMPLETE THE ROW REDUCTION.

b) The Gaussian algorithm results in the following augmented matrix. Write down the general solution to the system of equations.

$$\begin{bmatrix} 1 & -2 & 0 & 0 & | & 0 \\ 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 1 & | & 0 \end{bmatrix}$$

c) Let U be the set of all solutions to the above system of equations. Explain briefly why U is a subspace of $\mathbf{R}^4$.

d) Let A be the coefficient matrix of the above system of equations.
   * What is the rank of A?
   * Does $Ax = b$ have a solution for every $b \in \mathbf{R}^3$?
   * Explain how these two questions are related.

14. Consider the vectors $v_1 = (1, -2, -4)$, $v_2 = (2, -3, 1)$, and $v_3 = (3, -5, -3)$. Justify your answer to each of the following questions.

a) Does $\{v_1, v_2, v_3\}$ span $\mathbf{R}^3$?

b) Is $\{v_1, v_2, v_3\}$ linearly independent?

c) Let V be the subspace of $\mathbf{R}^3$ generated by the vectors v_1, v_2 and v_3. What is the dimension of V? Give a geometric description of V.