University of Ottawa
Department of Mathematics and Statistics

Alistair Savage

Undergraduate Research Opportunities – 2017

Overview. The projects below are at a level appropriate for undergraduates. This is not a complete list, but rather a representative sample of the type of projects that can be undertaken. They can be completed as part of an NSERC Undergraduate Summer Research Award (USRA), Undergraduate Research Opportunity Program (UROP), Work-Study Program, Co-op Program, or Undergraduate Research Project (MAT 4900). Undergraduate students interested in learning more about these and related projects should contact me. Projects completed by past students can be found at

`http://alistairsavage.ca/students`.

Successful projects could lead to publication. (See the above link for examples of undergraduate projects that have been published.) All of the projects below lead naturally to M.Sc. theses, for those students interested in pursuing graduate studies.

Prerequisites. Students interested in working on these projects should have a background in linear algebra (MAT 1341, 2141) and group theory (MAT 2143).

Projects

Formal group laws. A formal group law is a power series $F(x, y)$ in two variables such that

- $F(x, y) = x + y +$ terms of higher degree,
- $F(x, (F(y, z))) = F(F(x, y), z)$ (associativity).

One should think of F as something like the formal power series expansion of a product of a Lie group. The simplest examples of formal group laws are

- $F(x, y) = x + y$ (the additive formal group law), and
- $F(x, y) = x + y + xy$ (the multiplicative formal group law).

One can show that formal group laws also have formal identity elements and formal inverses. The theory of formal group laws is very rich and has connections to algebra, geometry, topology, number theory, and Lie theory. The goal of this project is to develop a treatment of formal group laws using the language of category theory that is accessible to undergraduate students.

The Farahat–Higman ring. To any group G, one can associate the *group ring* $\mathbb{Z}G$, which is the set of formal $\mathbb{Z}$-linear combinations of elements of G, with multiplication given by linearity and the group operation. The *center* of a ring is the set of all elements of the ring that commute with every other element. One can take a certain limit of the centers of the group rings $\mathbb{Z}S_n$ of the symmetric groups S_n. This limit is called the *Farahat–Higman ring*. It plays an important role in the representation theory of symmetric groups. Recent research has shown that the Farahat–Higman ring has a natural realization in terms of planar braid-like diagrams. The goal of this project is to describe this realization precisely, drawing from results in the literature. Time permitting, the student could also develop a so-called "quantization" of this ring.

Operads. An operad is a model for certain properties such as commutativity, anticommutativity, or associativity. They can be viewed as a set of operations that can be composed with one another, and are related to universal algebra. Algebras (which generalize the concept of a ring) are to operads as group representations are to groups. Operads are important tools in algebraic topology and graph theory. The goal of this project is to investigate the theory of operads, surveying the known results and investigating some new operads not previously considered.