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Undergraduate Research Opportunities

Overview. The projects below are at a level appropriate for undergraduates. This is not a complete list, but rather a representative sample of the type of projects that can be undertaken. They can be completed as part of an NSERC Undergraduate Summer Research Award (USRA), Undergraduate Research Opportunity Program (UROP), or Undergraduate Research Project (MAT 4900). Undergraduate students interested in learning more about these and related projects should contact me. Projects completed by past students can be found at

https://alistairsavage.ca/students.

Successful projects could lead to publication. (See the above link for examples of undergraduate projects that have been published.) All of the projects below lead naturally to M.Sc. theses, for those students interested in pursuing graduate studies.

Prerequisites. Students interested in working on these projects should have a background in linear algebra (MAT 1341, 2141) and group theory (MAT 2143). A background in ring theory (MAT 3143) would also be an asset.

Projects

Hopf algebras. The concept of an associative algebra over a commutative ring is a generalization of the notation of a ring. Precisely, a ring is an associative algebra over $\mathbb{Z}$. A Hopf algebra is an associative algebra with some additional structure. This structure is *precisely* what one needs to construct tensor products of modules, trivial modules, and dual modules. Hopf algebras play an important role in representation theory as well as in knot theory. In particular, they can be used to construct invariants of braids and links.

The goal of this project is to give an overview of Hopf algebras accessible to undergraduate students, with a special focus on motivation and applications. Time permitting, you can explore *super* versions of Hopf algebras.

Idempotent completions. The *idempotent completion*, or Karoubi envelope, of a category is a way of formally adding in "missing objects" to a category. The idea is that every *idempotent morphism* (a map in a category that squares to itself) should correspond to a projection. If it does not, then we formally add an object to our category so that the idempotent is projection onto the new object. Idempotent completions play an important role in modern representation theory. In particular, they have nice universal properties.

The goal of this project is to give a nice exposition of idempotent completions, together with an overview of their importance in modern mathematics. Time permitting, you can look at idempotent completions in 2-categories.

Symmetric functions. Symmetric functions are essentially symmetric polynomials (i.e. polynomials left unchanged under permutation of the variables) in infinitely many variables. They have deep connections to many areas of algebra, geometry, and combinatorics. Laurent symmetric functions are obtained from usual symmetric functions by making the variables invertible. The goal of this project is to study the theory of Laurent symmetric functions and explore which results for usual symmetric functions hold for Laurent symmetric functions.

Frobenius 0-Hecke algebras. Hecke algebras play a crucial role in the representation theory of Lie algebras and quantum groups. They depend on a quantum parameter q, and specializing this parameter to $q = 0$ produces what are known as 0-Hecke algebras. 0-Hecke algebras have connections to representation theory, combinatorics, and the geometry of the flag variety. Recently, the definition of 0-Hecke algebras has been generalized to incorporate a Frobenius algebra. (This was part of a previous student project that can be found here.) The case where this Frobenius algebra is the ground field recovers the standard 0-Hecke algebras. The goal of this project is to continue this investigation, generalizing more of the theory of 0-Hecke algebras. This would involve working with string diagrams in monoidal categories and would be an analogue of a former USRA project, involving nilHecke algebras, that can be found here (or as a preprint here).