University of Ottawa

Department of Mathematics and Statistics

Alistair Savage

Undergraduate Research Opportunities – 2018

Overview. The projects below are at a level appropriate for undergraduates. This is not a complete list, but rather a representative sample of the type of projects that can be undertaken. They can be completed as part of an NSERC Undergraduate Summer Research Award (USRA), Undergraduate Research Opportunity Program (UROP), Work-Study Program, Co-op Program, or Undergraduate Research Project (MAT 4900). Undergraduate students interested in learning more about these and related projects should contact me. Projects completed by past students can be found at

http://alistairsavage.ca/students.

Successful projects could lead to publication. (See the above link for examples of undergraduate projects that have been published.) All of the projects below lead naturally to M.Sc. theses, for those students interested in pursuing graduate studies.

Prerequisites. Students interested in working on these projects should have a background in linear algebra (MAT 1341, 2141) and group theory (MAT 2143).

Projects

Super monoidal categories. A monoidal category is a category that is equipped with a "tensor product". Some examples include:

- sets with the Cartesian product,
- vector spaces with the tensor product of vector spaces,
- rings with the tensor product of rings.

Super mathematics involves the introduction of parity to various fields of mathematics. For example, super vector spaces are vector spaces V that decompose as a sum $V = V_0 \oplus V_1$, where the elements of V_0 are said to be *even*, and the elements of V_1 are said to be *odd*. There are well-developed theories of associative superalgebras, Lie superalgebras, and supermanifolds, in addition to super variants of other fields of mathematics.

The purpose of this project is to write down the precise definitions of *super monoidal categories*. These have appeared implicitly in the literature, but cannot be found in standard textbooks. Thus, you would be extracting the definitions from research papers and writing a clear self-contained exposition of the subject.

Idempotent completions. The *idempotent completion*, or Karoubi envelope, of a category is a way of formally adding in "missing objects" to a category. The idea is that every *idempotent morphism* (a map in a category that squares to itself) should correspond to a projection. If it does not, then we formally add an object to our category so that the idempotent is projection onto the new object. Idempotent completions play an important role in modern representation theory. In particular, they have nice universal properties.

The goal of this project is to give a nice exposition of idempotent completions, together with an overview of their importance in modern mathematics. Time permitting, you can look at idempotent completions in 2-categories.

Operads. An operad is a model for certain properties such as commutativity, anticommutativity, or associativity. They can be viewed as a set of operations that can be composed with one another, and are related to universal algebra. Algebras (which generalize the concept of a ring) are to operads as group representations are to groups. Operads are important tools in algebraic topology and graph theory. The goal of this project is to investigate the theory of operads, surveying the known results and investigating some new operads not previously considered.